Lodz University of Technology
Department of Automation, Biomechanics and Mechatronics

OPEN SCIENTIFIC LECTURES IN K11 2025

On the Relationships Between Friction-Induced Acoustic Waves and Vibrations: A Review with an Illustrative Experiment

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13. May 2025, 12:15, Room 2M334

Introduction to Asperity Contact in Frictional Interfaces

- **Asperities** are microscopic surface peaks that form the actual contact points between rough surfaces.
- Contact occurs only at asperity tips.
- These micro-contacts experience high local pressure, influencing:
  - Frictional resistance
  - Acoustic wave generation
  - Vibration behavior
  - Surface wear
- Understanding asperity interactions is fundamental to analyzing friction-induced acoustic and vibrational phenomena.

Modeling Asperity Contact – Bhushan's Approach

- Bhushan builds on the Greenwood-Williamson model, treating rough surfaces as a collection of **spherical asperities** with statistical height distribution.
- Contact occurs only when asperity height *z* exceeds the separation distance *d*.
- The real area of contact and total normal load are computed by integrating Hertzian contact relations over all contacting asperities.

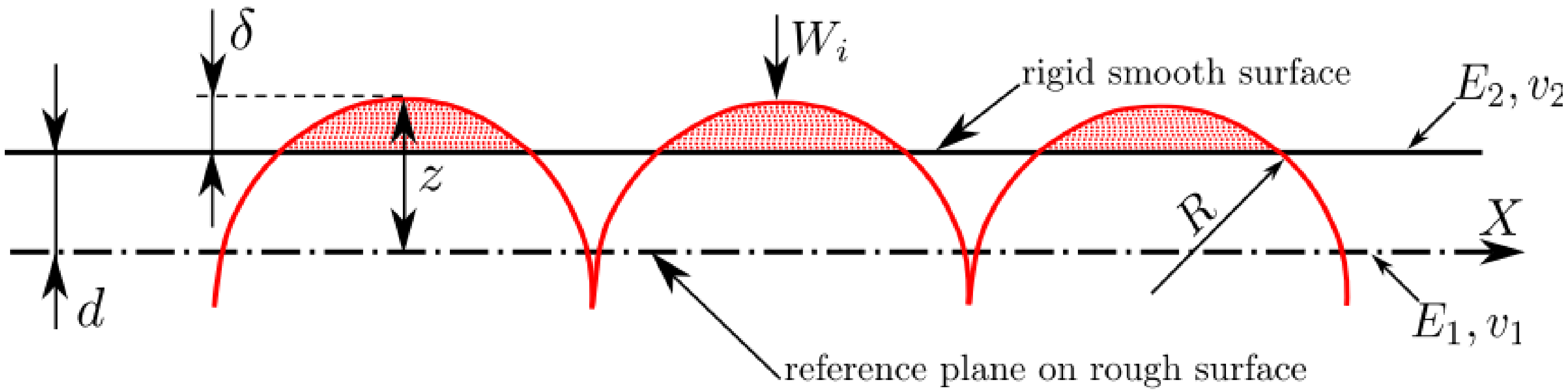

Fig. 1 Schematic of contact of a regular patterned rough surface against a smooth plane surface [1]

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Key Equations:

- Deformation (indentation depth):

$$\delta = z - d$$

- Contact area per asperity:

$$A_i = \pi R \delta$$

- Normal load per asperity (Hertzian contact):

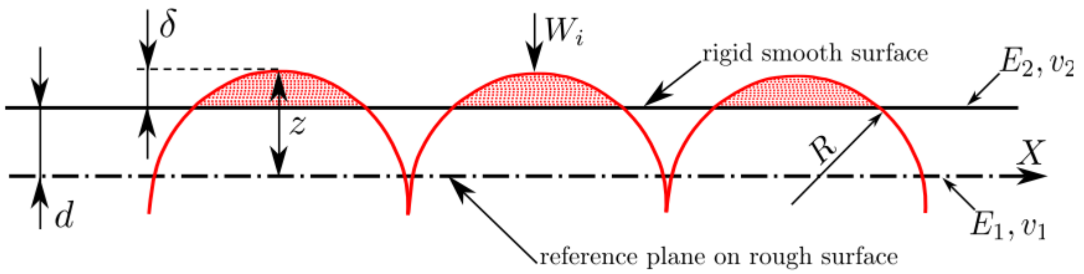
$$W_i = \frac{4}{3} E^* R^{1/2} \delta^{3/2}$$

- Total real contact area:

$$A_{re} = \pi N R \int_d^{\infty} (z - d) p(z) dz$$

- Total normal load:

$$W = \frac{4}{3} N E^* R^{1/2} \int_d^{\infty} (z - d)^{3/2} p(z) dz \qquad (1)$$



Parameter Definitions:

- R : Radius of curvature of each asperity
- N : Total number of asperities on the surface
- E^* : Effective elastic modulus of the contacting bodies
- z : Asperity height above the reference plane
- d : Separation distance between surfaces
- $p(z)$: Probability density function of asperity heights
- δ : Local indentation depth $(z - d)$

Dynamics of Contact and Frictional Forces

- Friction arises from the deformation of surface asperities under load and relative motion.
- As asperities resist or assist motion, they generate transient forces that **excite vibrations** in the surrounding structure.

These vibrations can radiate as **acoustic emissions (AE)** — elastic waves detectable as sound or stress pulses.

Asperity Contact Kinematics:

$$-\dot{h}_n^i + \dot{s}_t^i = \dot{\delta}_t a_t^i + \dot{\delta}_n a_n^i, \qquad i = 1, \ldots, k \qquad (2)$$
$$-\dot{h}_n^j - \dot{s}_t^j = \dot{\delta}_t a_t^j + \dot{\delta}_n a_n^j, \qquad j = 1, \ldots, l$$

Where,

$\dot{h}_n, \dot{s}_t$: Relative surface velocities (normal and tangential)

$\dot{\delta}_n, \dot{\delta}_t$: Asperity deformation rates . a_n, a_t : Geometric projections in the normal and tangential directions. k, l : Number of asperities resisting or assisting motion

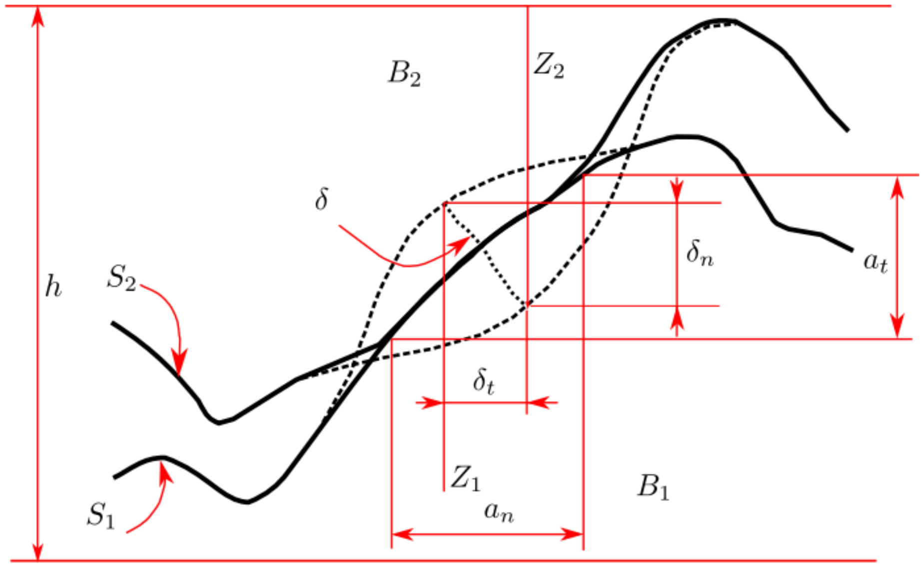


Fig. 2 Cross-section of a typical asperity contact showing projections of deformation and contact areas. Solid lines represent deformed surfaces; dotted lines indicate undeformed surfaces [2].

Sound Pressure Level

Sound pressure level (SPL) is the principal parameter used in airborne acoustic measurements, quantifying sound intensity relative to a reference pressure.

Acoustic Correlations:

- Sound pressure level (SPL):

$$\mathrm{SPL}(\,\mathrm{dB}) = 20\log_{10}\left(\frac{p_{\mathrm{RMS}}}{p_0}\right) \qquad (3)$$

where:

- p_{RMS} : Root mean square pressure in (Pa)
- p_0: Reference Sound Pressure, typically 20μ Pa $= 2 \times 10^{-5}$ Pa** in air, which is the standard threshold of human hearing.

Effect of Particles on Contact and Acoustic Behavior

- Ghaednia and Jackson proposed a multiscale model integrating rough surface contact and statistical particle mechanics to evaluate friction, wear, and acoustic activity.

- Total friction coefficient:

$$\mu = \frac{A_s\tau_s + A_p\tau_p}{F_{\text{ext}}} \quad (4)$$

Where:

A_s, A_p : Contact area of surface and particles,

τ_s, τ_p : Shear stress on surface asperities and particles,

F_{ext} : Applied normal force.

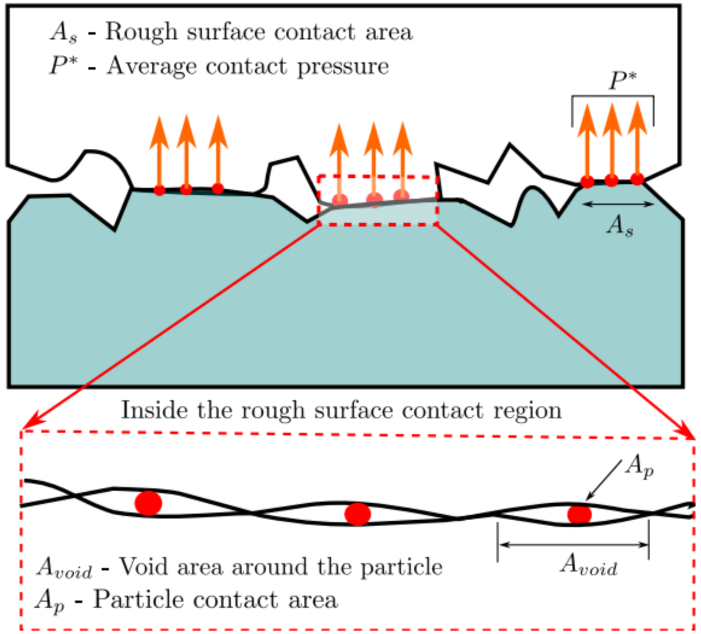


Fig. 3 Illustration of a stacked rough surface and statistical model [3].

Nonlinear Vibration and Acoustic in the Brake System

- Soobbarayen developed a nonlinear model of disc brake systems to analyze vibration instabilities and resulting acoustic radiation.
- The model uses a disc–pad contact configuration, governed by Coulomb friction law, and includes both linear and nonlinear stiffness components.

Friction Force Definitions:

$$\begin{cases} F^{d}_{friction,\ x} = \mu F^{d}_{contact\ ,z}\,\mathrm{sign}(v_r)\mathbf{e}_\theta \cdot \mathbf{x} \\ F^{d}_{friction,\ y} = \mu F^{d}_{contact\ ,z}\,\mathrm{sign}(v_r)\mathbf{e}_\theta \cdot \mathbf{y} \end{cases}$$

$$\begin{cases} F^{p}_{friction,\ x} = -F^{d}_{friction,\ x} \\ F^{p}_{friction,\ y} = -F^{d}_{friction,\ y} \end{cases}$$

$$m\ddot{x} + c\dot{x} + kx = F_{NL} + F. \qquad (9)$$

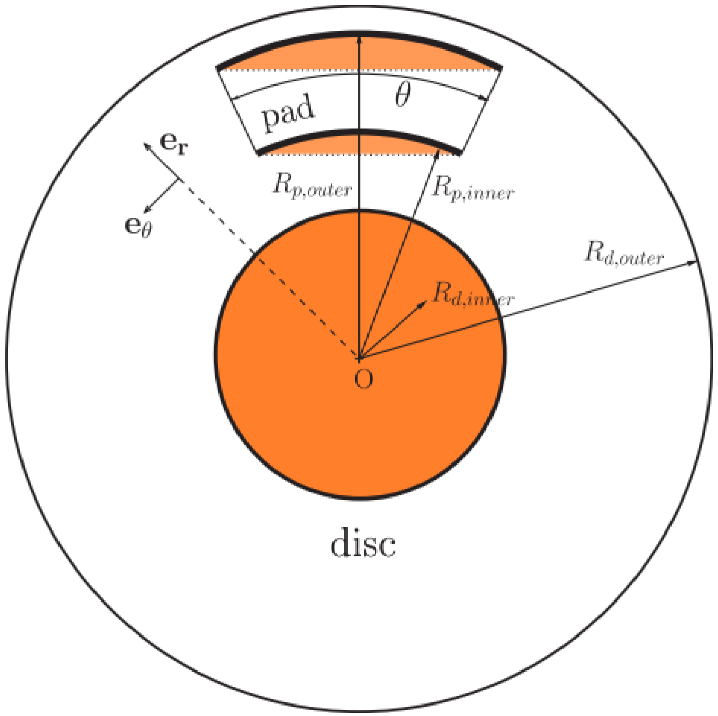


Fig. 6 Illustration of the geometrical parameters [7]

Pad–Disc Analysis

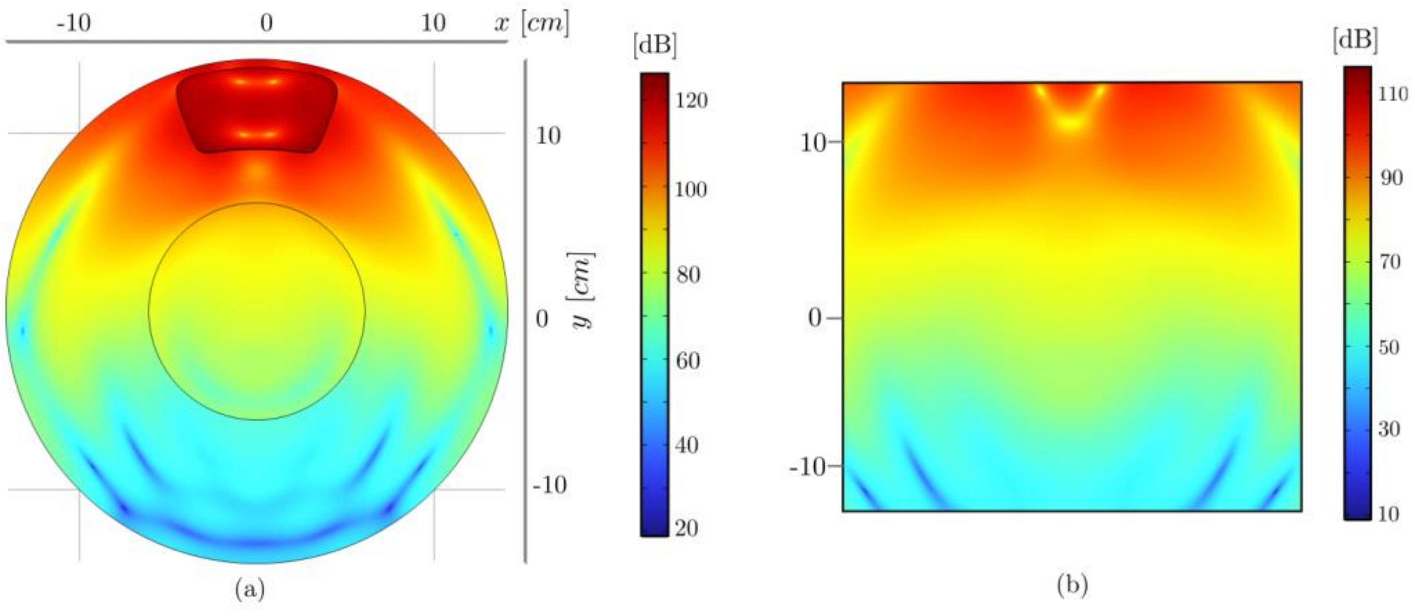


Fig. 7 Acoustic intensity estimated in COMSOL during stationary regime for μ = 0.72 at a frequency of 6426 Hz. (a) Disc and pad (BEM); (b) near field

Friction-Induced Vibrations in Beam–Surface Interaction

- The interaction between a vibrating beam and a flat surface under frictional contact leads to the generation of **longitudinal and bending waves**.

- A beam inclined at angle φ impacts the surface intermittently, causing discrete force excitations that generate broadband acoustic responses.

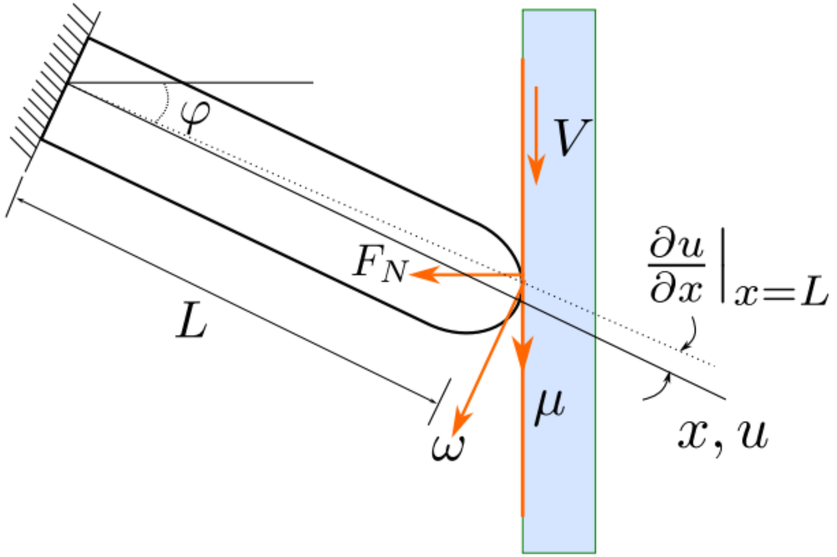


Fig. 4 Schematic figure of the rod and the disk [4]

Friction-Induced Vibrations in Beam–Surface Interaction

Governing Equations of Motion *(neglecting shear and rotational inertia)* [5,6]

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} + C\frac{\partial w}{\partial t} = F_N(t)[\mu(v_r)\cos\varphi + \sin\varphi]\delta(x-L), \quad (5)$$

$$EA\frac{\partial^2 u}{\partial x^2} - \rho A\frac{\partial^2 u}{\partial t^2} - C'\frac{\partial u}{\partial t} = F_N(t)[\cos\varphi - \mu(v_r)\sin\varphi]\delta(x-L). \quad (6)$$

Here, $w(x,t)$ and $u(x,t)$ represent the transverse and longitudinal displacements of the beam, respectively. The coefficient $\mu(v_r)$ is the velocity-dependent friction coefficient, and $F_N(t)$ is the time-varying normal contact force applied at the beam tip $(x=L)$. The material and geometric properties of the beam include density ρ, Young's modulus E, cross-sectional area A, and second moment of area I. The constants C and C' are damping coefficients proportional to the transverse and longitudinal vibration velocities, respectively.

For instance: The distribution of the Sound Pressure Level (dB) across the beam and disc surfaces for the model depicted in Fig. 4, with parameters $L = 50$ cm, $\varphi = 60°$, $V = 0.01$ m/s, $\mu = 0.3$ and $F_N = 10$ N, is shown in Fig. 5a and Fig. 5b for excitation frequencies of 750 Hz and 1250 Hz , respectively.

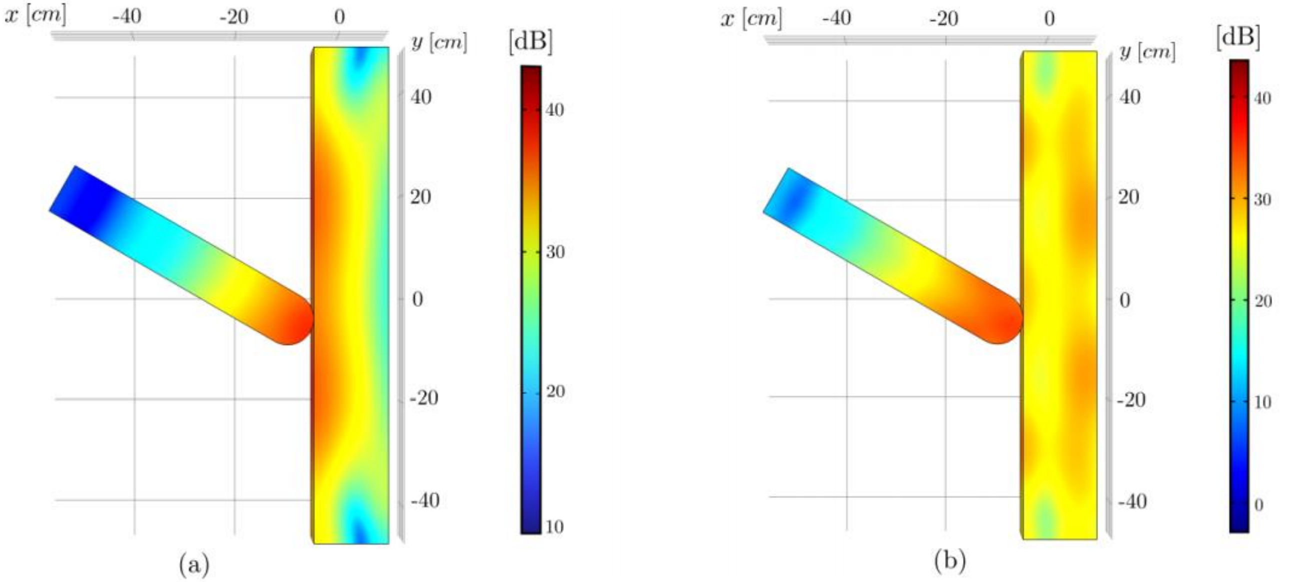


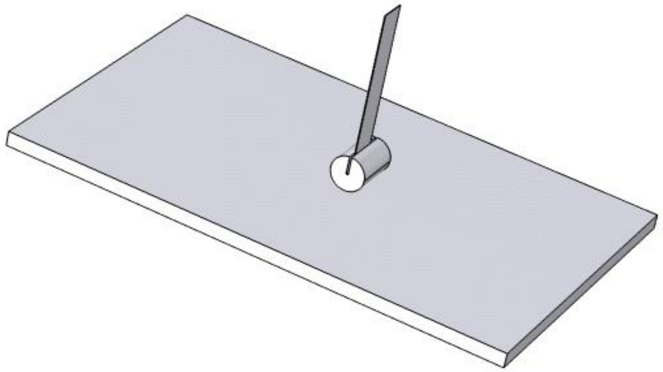
Fig. 5 Sound Pressure Level distribution estimated in COMSOL on the beam and disc surfaces at: (a) 750 Hz, (b) 1250 Hz

The acoustic wave model is described by a slightly modified form of the Helmholtz equation for acoustic pressure p :

$$\nabla \cdot \left(-\frac{\nabla p}{\rho} \right) - k^2 \frac{p}{\rho} = 0 \qquad (7)$$

Experimental illustration

- The experimental setup includes a table driven by a three-phase permanent magnet linear synchronous motor (PMLSM) with an LMCA4 inductor, an LMCAS3 magnetic track, and a Xenus XTL controller.



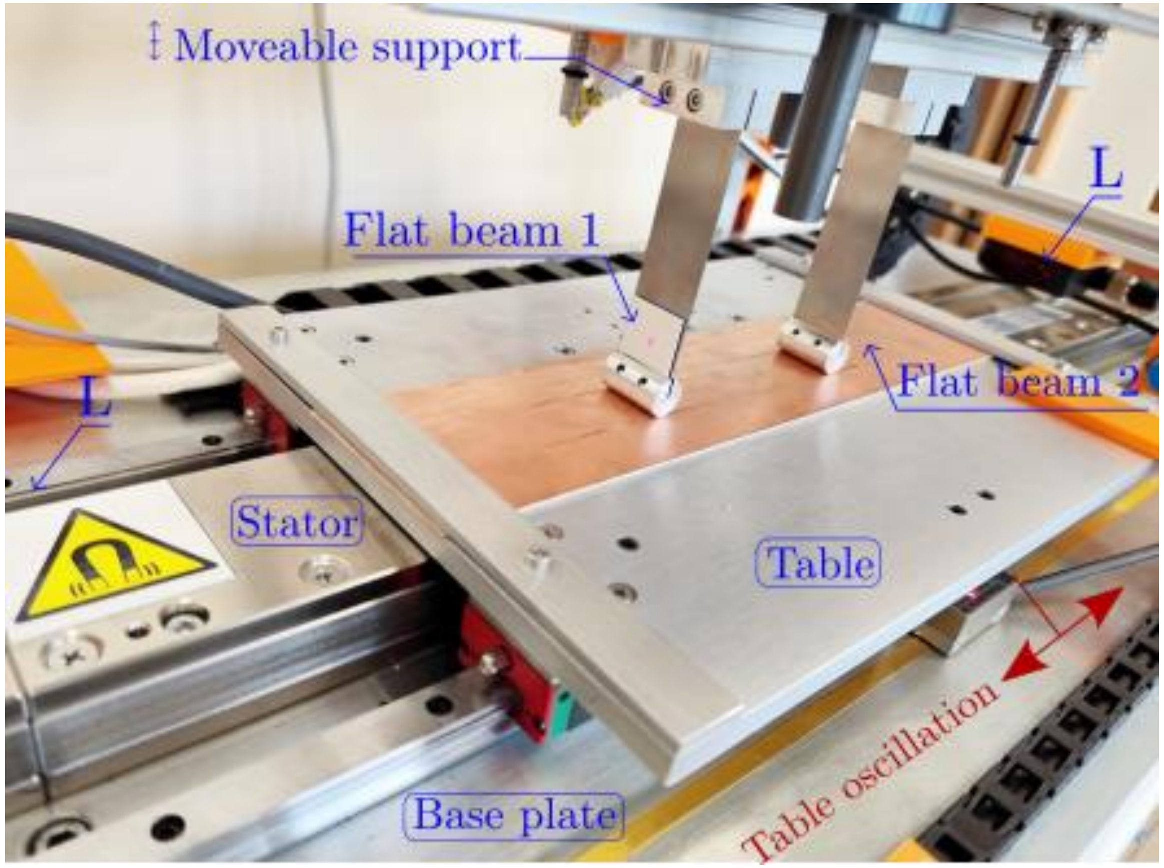


Fig. 8 Experimental station [Olejnik et al.,2025]

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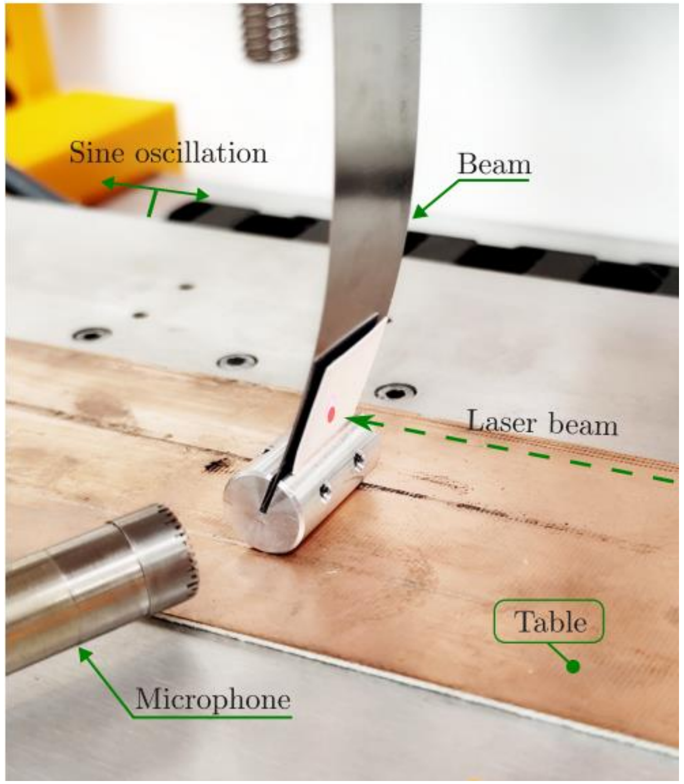

Fig. 9 The laboratory experiment with acoustic wave emission in friction-induced stick-slip vibrations in a 2-DOF mechatronic system consisting of a beam and table surface

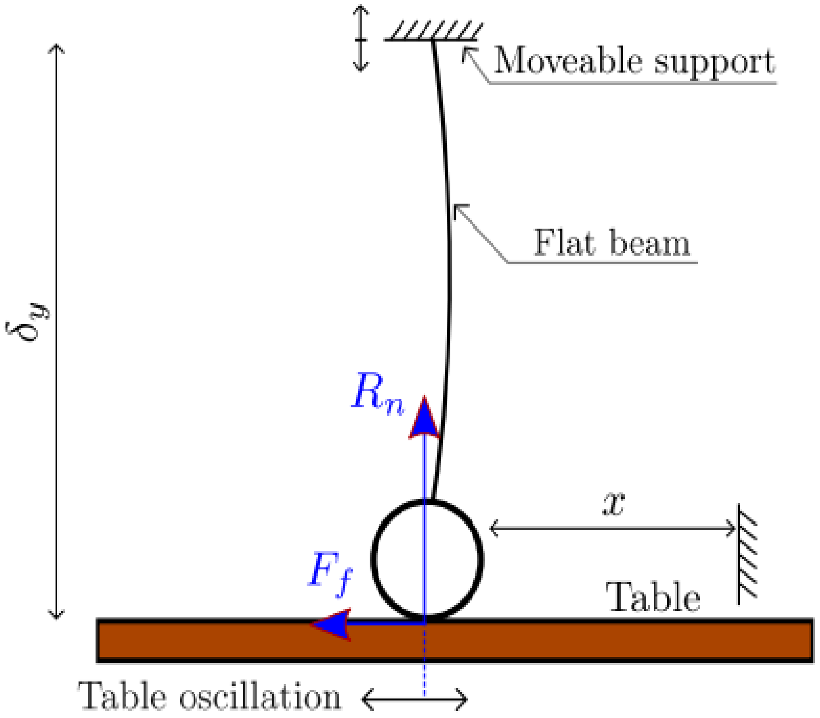

Fig. 10 Physical model of the table and the flexible flat beam dynamical interaction in a frictional contact (Rn – the normal reaction force, Ff – the surface frictional force).

Friction-induced vibration

- Mechanical phenomena such as stick-slip friction and acoustic emissions occur, especially during the quick buckling of flat beams.

- By selecting a cyclically changing table speed (e.g., quasi-sinusoidal motion) and adjusting either:

  - The force from the flat elastic element, or

  - The pair of friction surface materials, optimal conditions can be created for acoustic wave generation in the system.

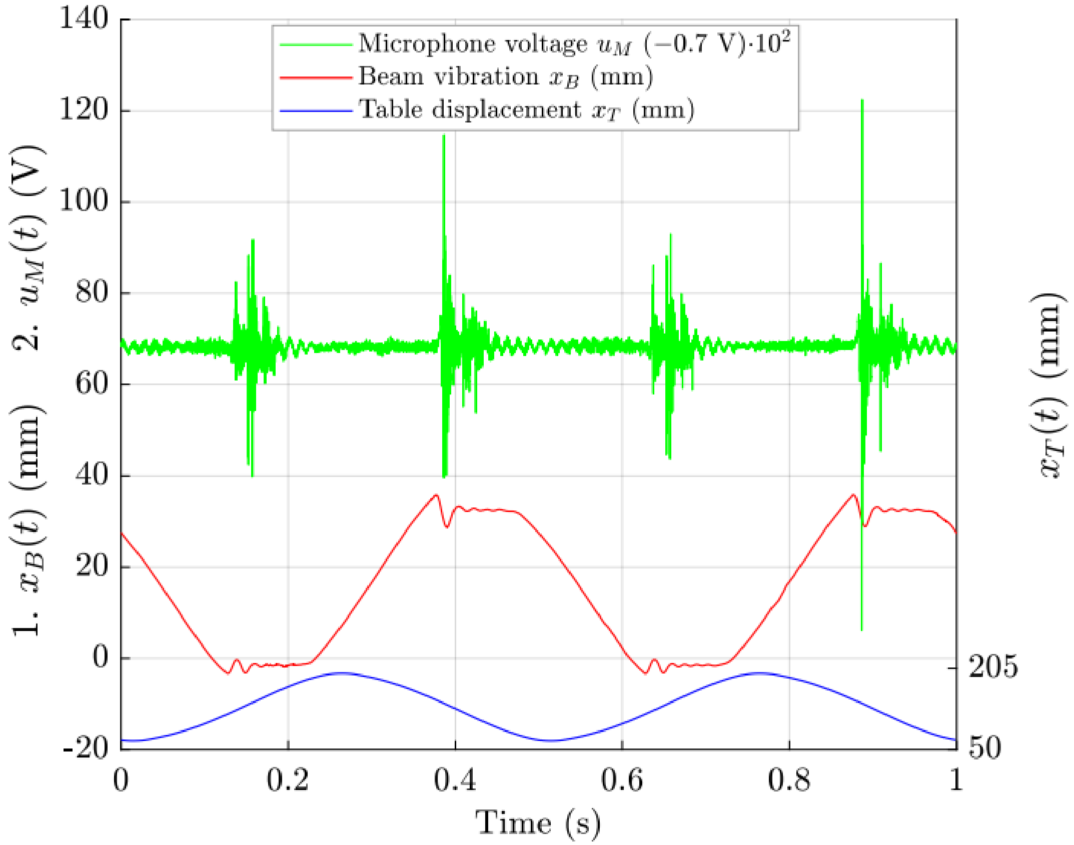

Fig. 11 The real friction-induced vibration profile with coexisting acoustic (rough) friction in a 2-DOF mechatronic system consisting of a beam and a table surface.

Stick-Slip Friction and Acoustic Emissions

- Synchronized measurements on the test stand reveal temporal relationships between cyclic stick-slip motion and acoustic friction (Fig. 11).
- During these events, brief acoustic wave emissions occur at varying frequencies.
- The spectral analysis (Fig. 12) identifies these emissions within a frequency range not exceeding the Nyquist limit of 5000 Hz.

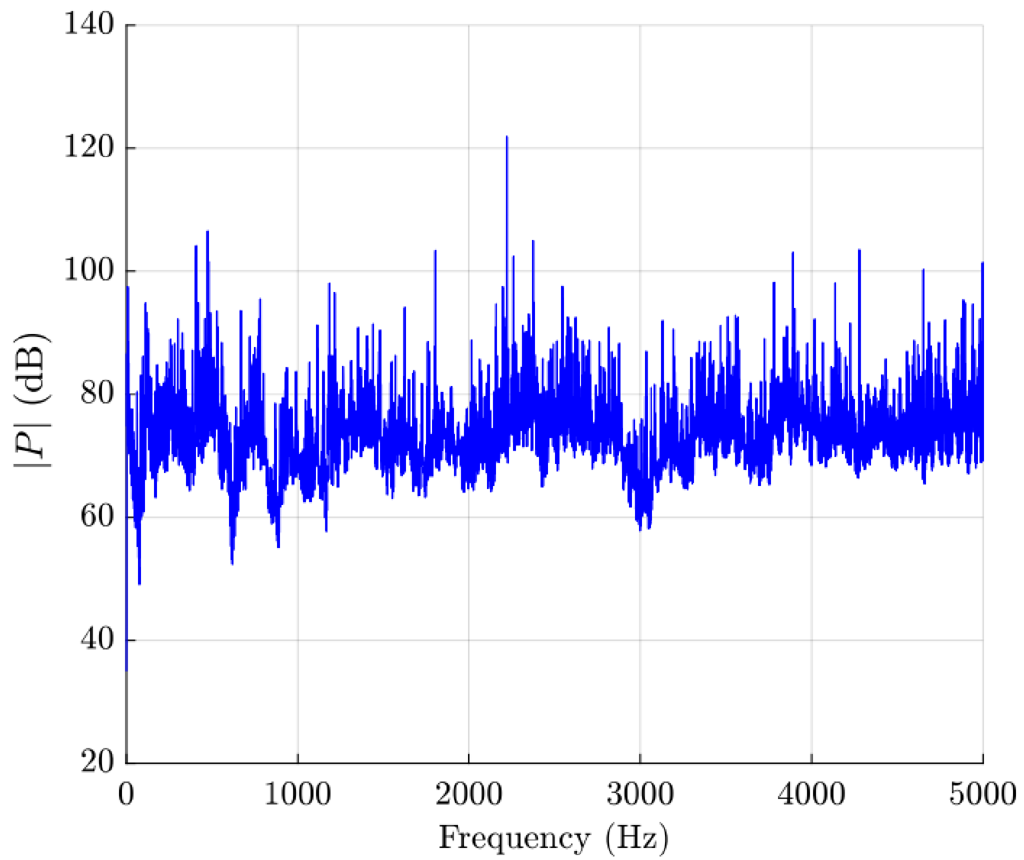

Fig. 12 Power spectrum of the microphone signal capturing rough friction in a 2-DOF mechatronic system consisting of a beam and a table driven by a PMLSM.

Implementation of the Exponential Coulomb Friction Law

- The analytical expression of the exponential Coulomb friction law defines the friction force variation as an exponential function of velocity.

$$F_f = \left(\mu_d + (\mu_s - \mu_d)\exp(-\alpha_{dc}\|V_s\|)\right)F_N \tag{10}$$

where μ_s is the static friction coefficient, μ_d denotes the dynamic friction coefficient, F_N denotes the normal force, α_{dc} represents the exponential decay coefficient, and V_s is the relative slip velocity between the two contact surfaces.

Table 1 Frictional contact parameters in numerical simulation

Parameter	symbol	Unit	Value
Static friction coefficient	μ_s	-	0.42
Dynamic friction coefficient	μ_d	-	0.32
Exponential decay coefficient	α_{dc}	s m^{-1}	0.01
Normal force	F_N	N	15.0
Relative slip velocity	V_s	m s^{-1}	0.11

FEM modeling

- In the simulation of the sound pressure level (SPL) the sinusoidal speed of the table is given by $v(t) = a\sin(\omega t)(\text{mms}^{-1})$, where $\alpha = 160$ mm and $\omega = 2\pi\text{rad/s}$, for the time $t \in [0,2]$s.
- The acoustic pressure on the surfaces is governed by Helmholtz equation for the acoustic pressure (p) which is presented as :

$$\nabla \cdot \left(-\frac{\nabla p}{\rho}\right) - k^2 \frac{p}{\rho} = 0 \qquad (11)$$

where $k = \omega/c$ defines the wave number, c is the speed of sound, ω is the angular frequency, and ρ is the density.

FEM Analysis

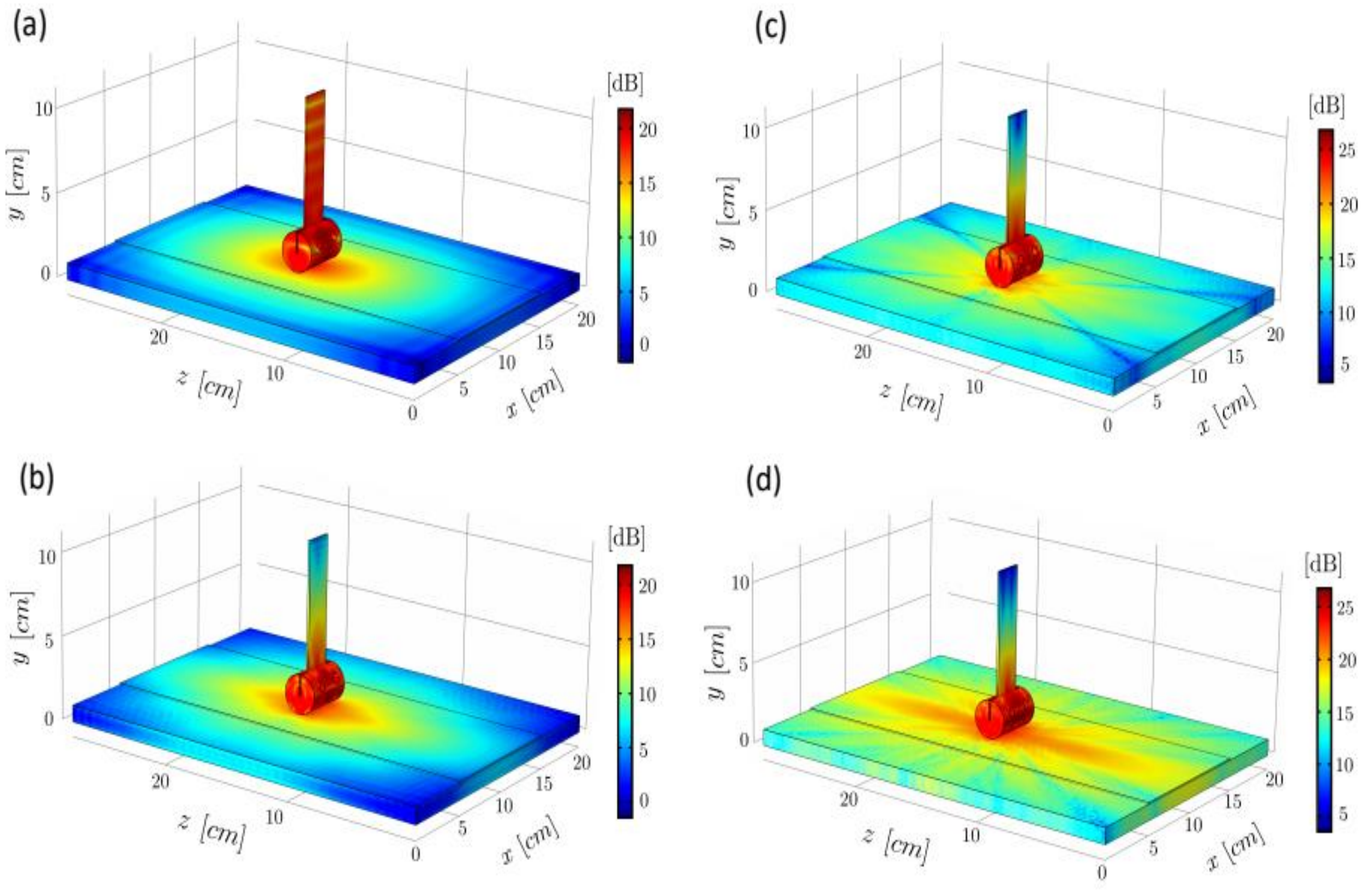

Fig. 13 Sound Pressure Level (dB) distribution on the beam and table surfaces at: (a) f_1 =100 Hz , (b) 2 f_1 , (c) 3 f_1 , and (d) 4 f_1

- The equation for sound level in decibels (dB) is:

$$L_{dB} = 20\log_{10}\left(\frac{P}{P_{\text{ref}}}\right) \qquad (12)$$

where:

- P = Measured sound pressure (Actual sound pressure level at a given time in Pa) .
- P_{ref} = Reference sound pressure (P_{ref} =$2e^{-5}$ Pa)

Conclusions

- Friction-induced vibrations are complex, coupled phenomena involving tribology, structural dynamics, and acoustics.
- Surface geometry, material properties, and dynamic loading critically influence energy dissipation, acoustic emission, and vibration.
- These factors are interdependent, with feedback loops leading to instabilities like stick-slip and mode coupling.
- Experimental and numerical results highlight multiscale interactions between vibration and acoustic emission, especially in stick-slip vibration.

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