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Estimating the static friction law of a kinematically forced double torsion pendulum using PINNs

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11. March 2025, 12:00, Room 2M334

1. INTRODUCTION

- In this work we introduce a physics-informed neural network approach for modeling friction, positioning it as an effective method for friction estimation.
- Experimental data from a double torsion pendulum system, featuring discontinuous dynamics, is utilized for training.
- The results highlight the network's superiority, providing a more precise representation of stick-slip phases at the contact zone.
- In summary, the presentation includes a case study showcasing the network's ability to predict dynamic models and estimate planar friction in a double torsion pendulum system, demonstrating its accuracy and efficiency.

2. MODELING THE INVESTIGATED DYNAMICAL SYSTEM

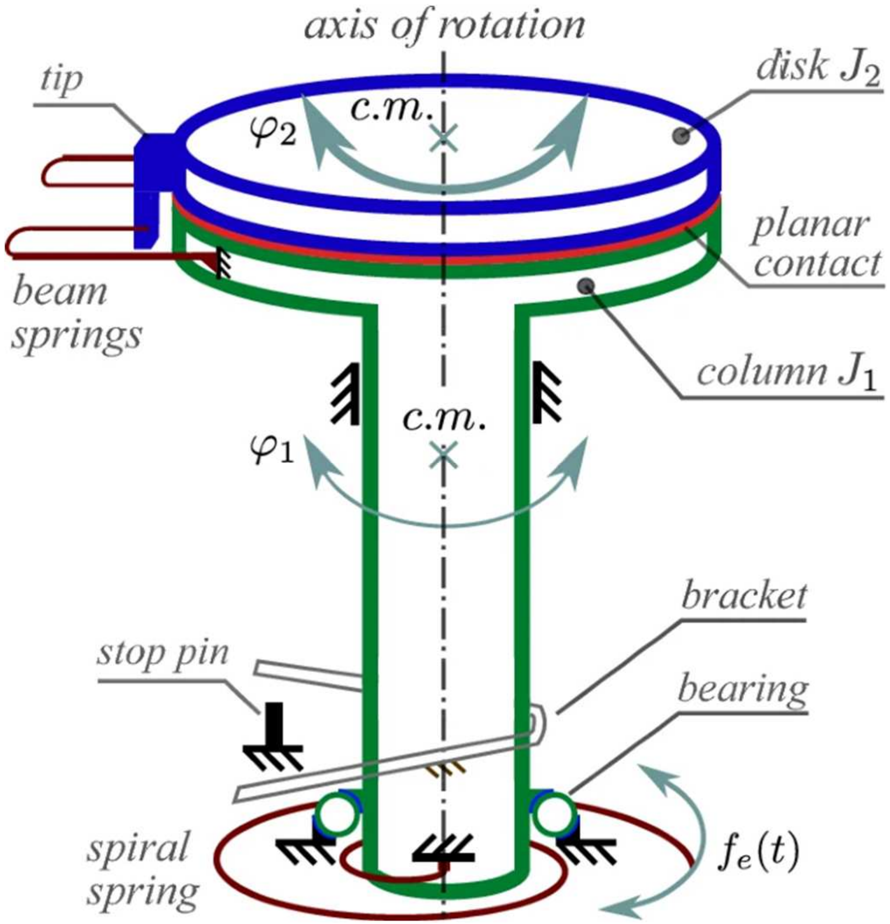


Fig. 1.

2. MODELING THE INVESTIGATED DYNAMICAL SYSTEM

The mathematical formulation of the simulated double torsion pendulum model was established in [35] using the Lagrange method. In the modified and reduced mathematical model, we introduce the time series $\varphi_{1,m}(t)$ as an input in [rad], representing the real angular displacement of the lower contact surface (illustrated in red in Fig. 1). Additionally, we calculate its approximate second time derivative. This leads us to the subsequent second-order semi-empirical dynamic system governing the rotational motion of the disk with respect to the φ_2 coordinate:

$$\frac{d^2\varphi_2}{dt^2} = \frac{1}{J_2}\left(-c_2\frac{d\varphi_2}{dt} - k_2\varphi_2 - F_f\right) - \frac{d^2\varphi_{1,m}}{dt^2}, \quad (1)$$

3. NUMERICAL SIMULATION OF THE MODEL

The four friction models that were simulated are the LuGre, Coulomb and viscous, Dahl and Coulomb, respectively.

The parameters of four experiments conducted in this experimental part are as follows: $J_2 = 2.17 \cdot 10^{-4}$, $c_2 = 0.19$, $k_2 = 0.7$, $\mu_v = 0.5$, $\mu_c = 0.12$, $\mu_s = 0.16$, $\beta = 2$, $\sigma_0 = 2 \cdot 10^4$, $\sigma_1 = 10^2$, $\sigma_2 = \mu_v$, $v_s = 10^{-3}$, and the parameter of smooth approximation of sign function, $\varepsilon = 10^3$. The initial conditions superposed on the disk body are zero while forcing of the mass is initiated by the input $d^2\varphi_{1,m}/dt^2$. More detail about the experimental part and its conditions can be found in [35].

3. NUMERICAL SIMULATION OF THE MODEL

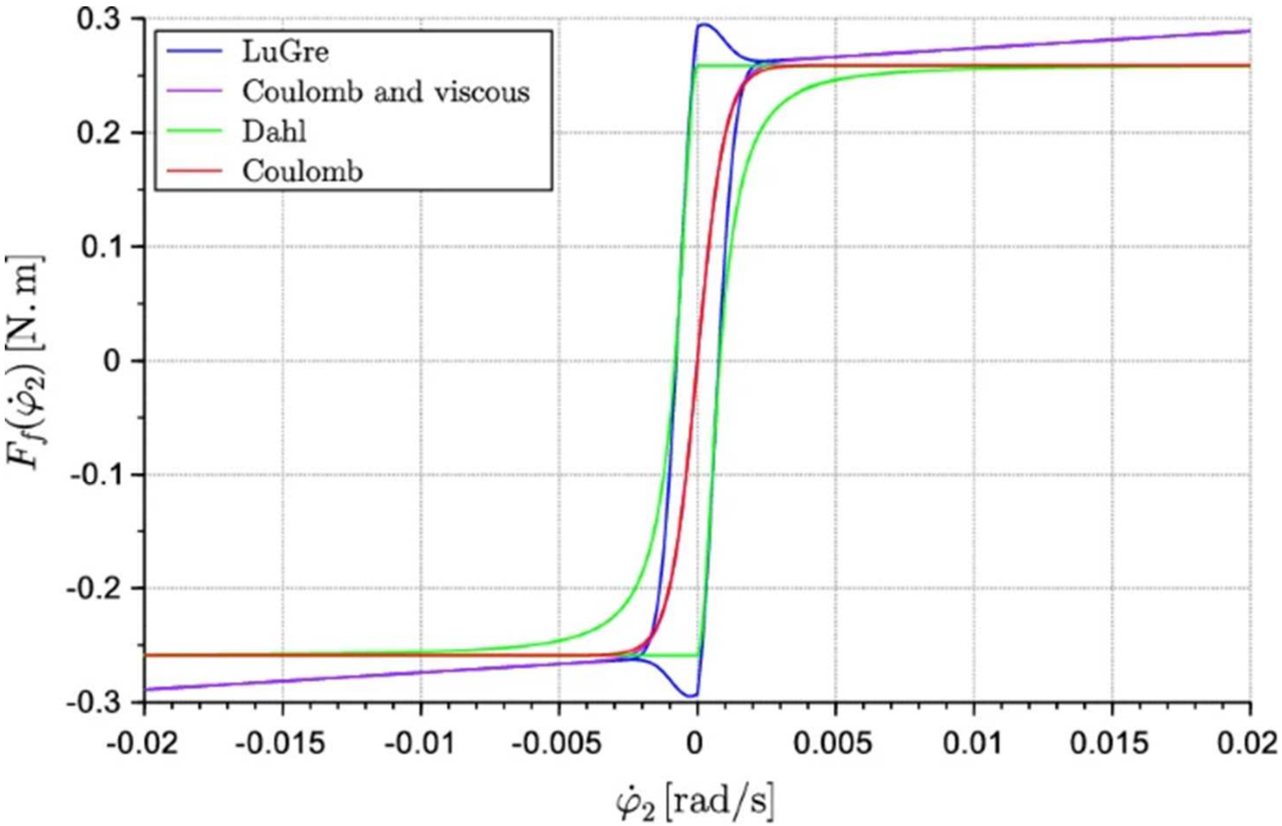


Fig. 2. (a) friction forces versus velocity

3. NUMERICAL SIMULATION OF THE MODEL

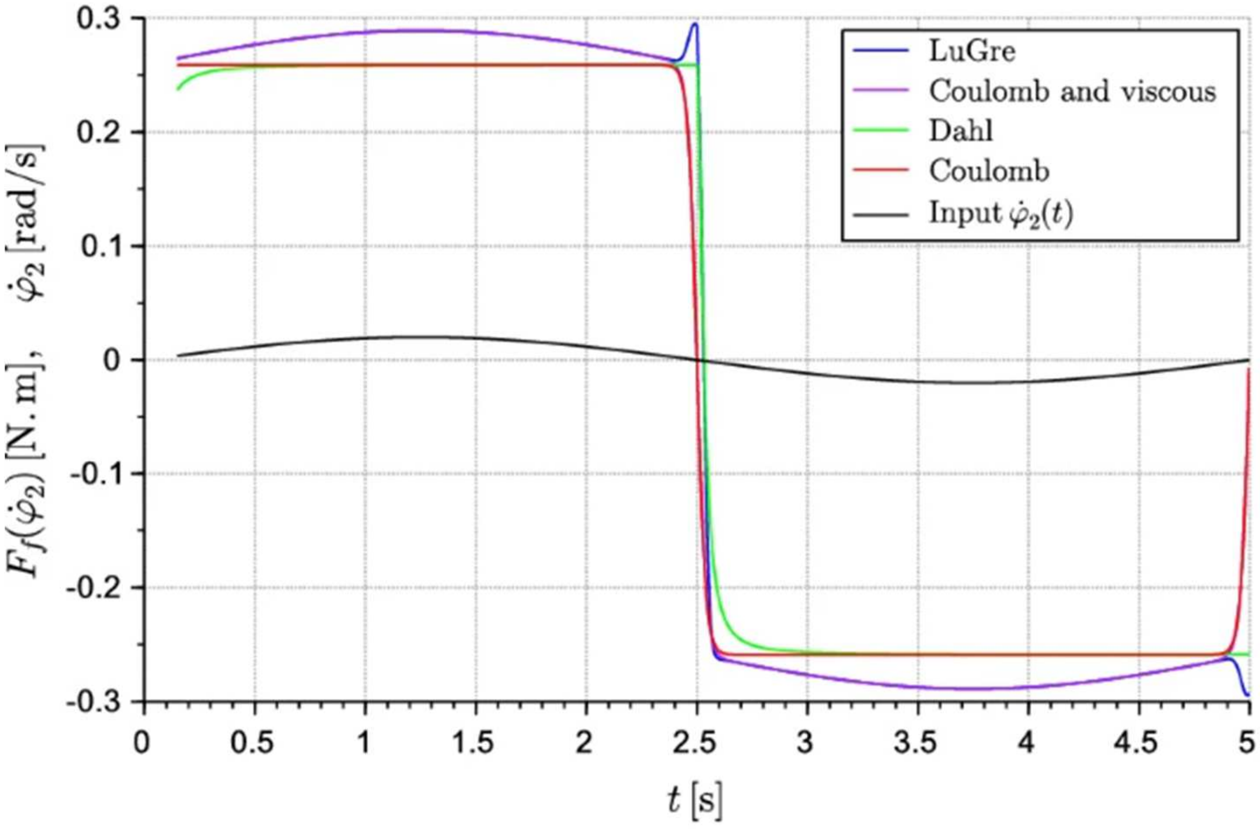


Fig. 3. (b) friction forces and input versus time

4. DATA-DRIVEN FRICTION MODELS

When dealing with translational mechanical or mechatronic systems, such as mass-spring systems pulled on a friction surface, the friction force and sliding velocity are measured to enable the estimation of the selected friction model. Conversely, in rotational mechanical or mechatronic systems, friction torque, and angular velocity are measured. At a constant velocity, the force or torque input to the system equals the friction force or torque (F_f or τ_f):

$$M\ddot{x} = F_{ap} - F_f \ . \tag{2}$$

When $\dot{x}$ is constant, $\ddot{x} = 0$ and $F_{ap} = F_f$. Similarly,

$$J\ddot{q} = \tau - \tau_f \ .$$

At constant angular velocity, $\ddot{q} = 0$ and $\tau_{ap} = \tau_f$, where F_{ap} and τ_{ap} are the applied friction and torque, respectively.

4. DATA-DRIVEN FRICTION MODELS

5.2 There are several standard concepts of data-driven modeling:

System excitation

Time-domain and frequency-domain data

Closed-loop and open-loop system identification

Online and offline identification

Data pre-processing

4. DATA-DRIVEN FRICTION MODELS

4.1 Black-box friction modelling

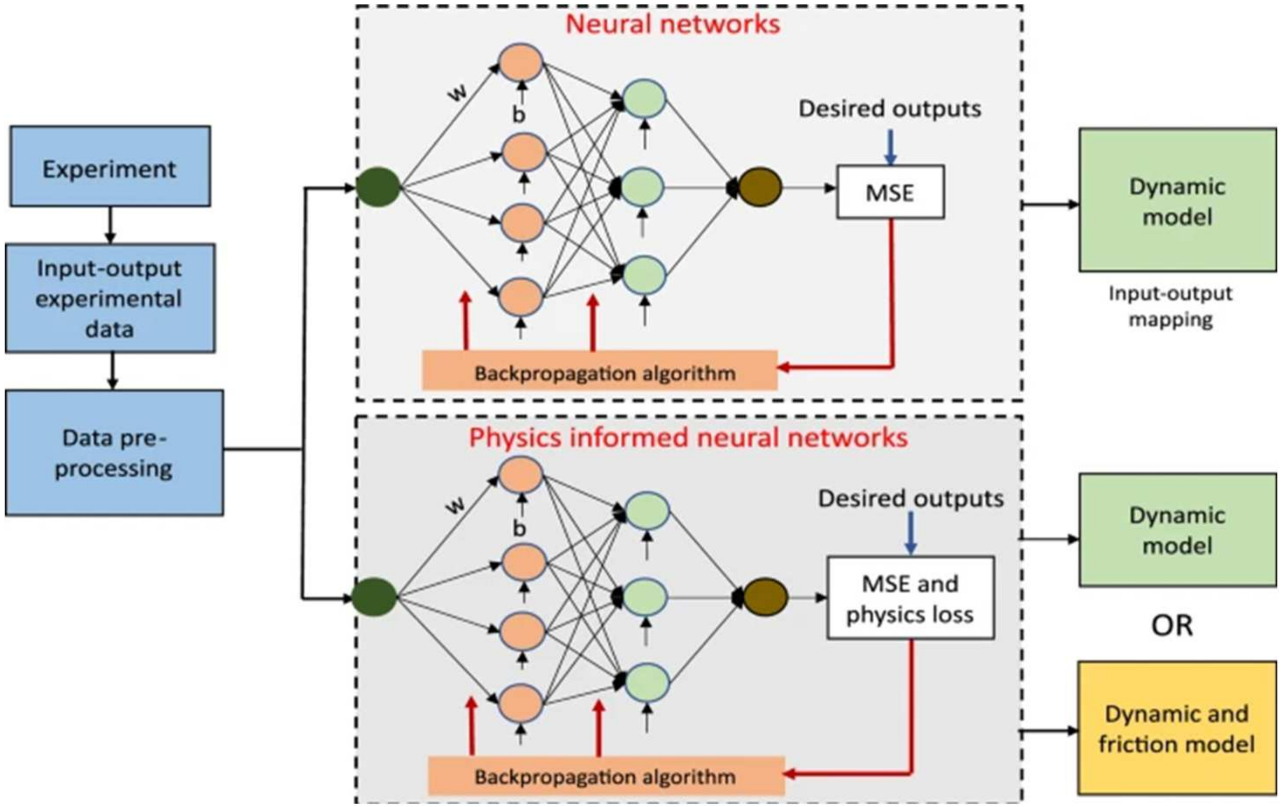


Fig. 4.

4. DATA-DRIVEN FRICTION MODELS

4.2 Black-box friction modeling – physics-informed neural networks (PINN)

PINN can be used to solve ordinary and partial differential equations [69, 70]. For example, given a first order differential equation below:

$$\frac{dy}{dt} = f(y, t, \gamma), \quad t \in [0, T] \tag{3}$$

where $y$is the dependent variable to be approximated by a neural network, t is time and γ denote the system parameter.

4. DATA-DRIVEN FRICTION MODELS

4.3 Black-box friction modeling – physics-informed neural networks (PINN)

The solution of the equation (i.e., y) can be approximated by a neural network: $\tilde{N}(t) \approx y(t)$. The derivative of the network output is computed with respect to its inputs through automatic differentiation. By virtue of the network differentiation, the original equation can be encoded into the loss function that is used in updating the weights and biases of the network.

$$L_{eq} = \frac{d\tilde{N}(t)}{dt} - f(\tilde{N}(t), t, \gamma) \quad (4)$$

4. DATA-DRIVEN FRICTION MODELS

As a result, the new loss function that is used to optimize the neural network is [71]:

$$L_T = L_s + L_{eq} \quad \textbf{(5a)}$$

$$L_T = \underbrace{\frac{1}{m}\sum_{i}^{m}\left(y(t_i) - \tilde{N}(t_i)\right)^2}_{Loss\ \text{of the solution}} + \underbrace{\frac{1}{m}\sum_{i}^{m}\left(\frac{d\tilde{N}(t_i)}{dt} - f(\tilde{N}(t_i), t_i, \gamma)\right)^2}_{Loss\ \text{of the equation}} \quad \textbf{(5b)}$$

where L_s is the loss of the solution, L_{eq} is the loss computed based on the system equation and L_T is the total loss.

5. THE ROTATIONAL CONTACT SURFACE: TORQUE ESTIMATION

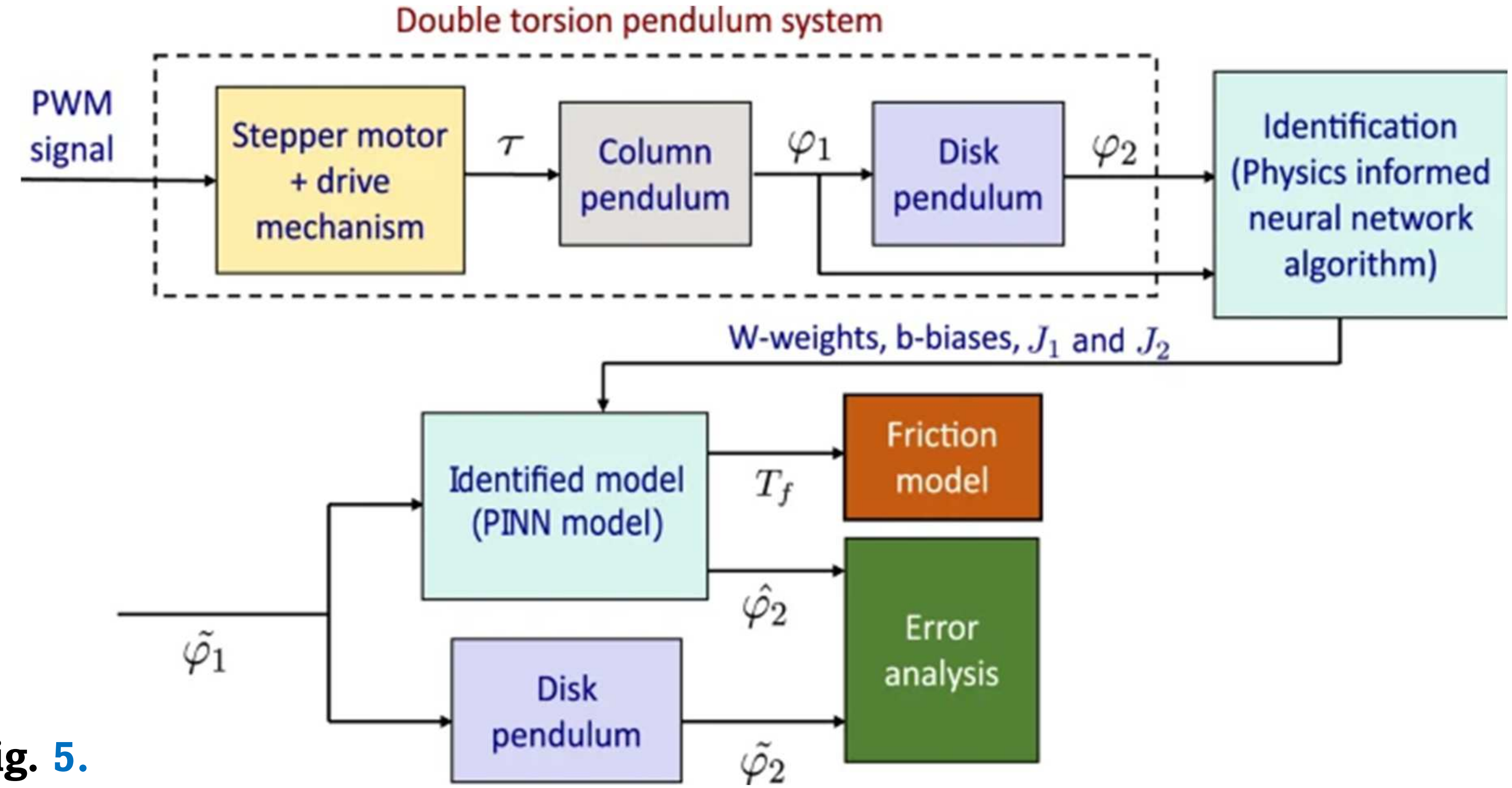


Fig. 5.

5.1 THE EXPERIMENTAL TEST STAND

1 – the disk

2 – stop pin

3 – frame

4 – elastic beams

5 – the column

6 – cam forcing the first end of the spiral spring attached to the column

7 – base

8 – microcontroller

9 – ball bearing

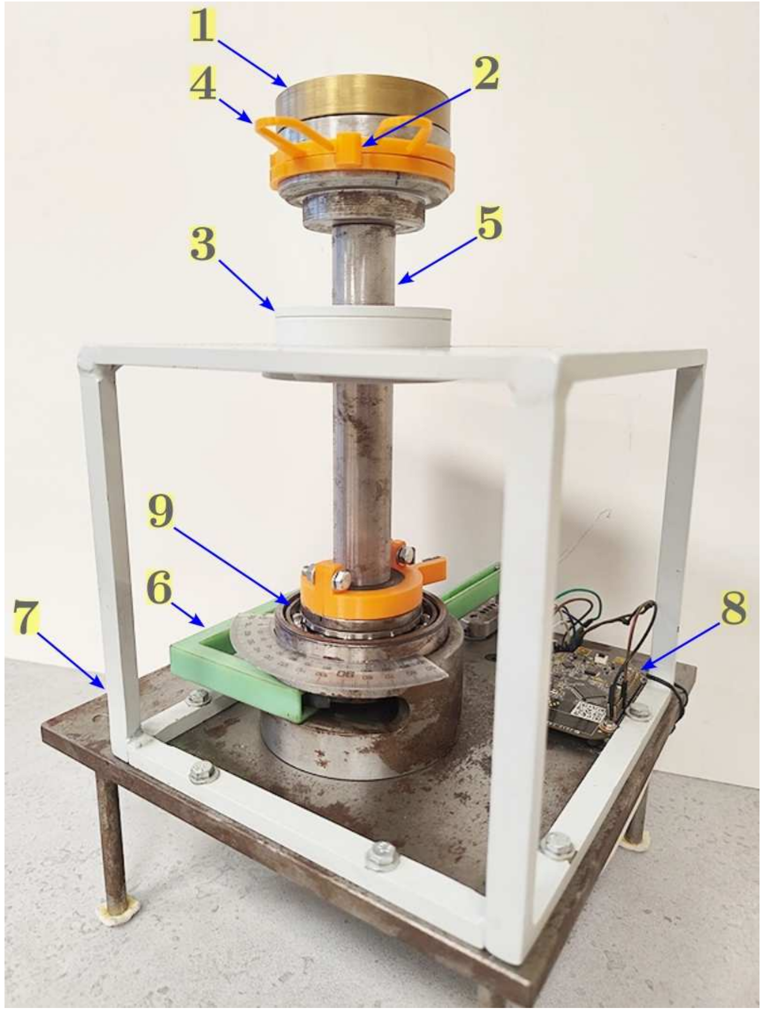
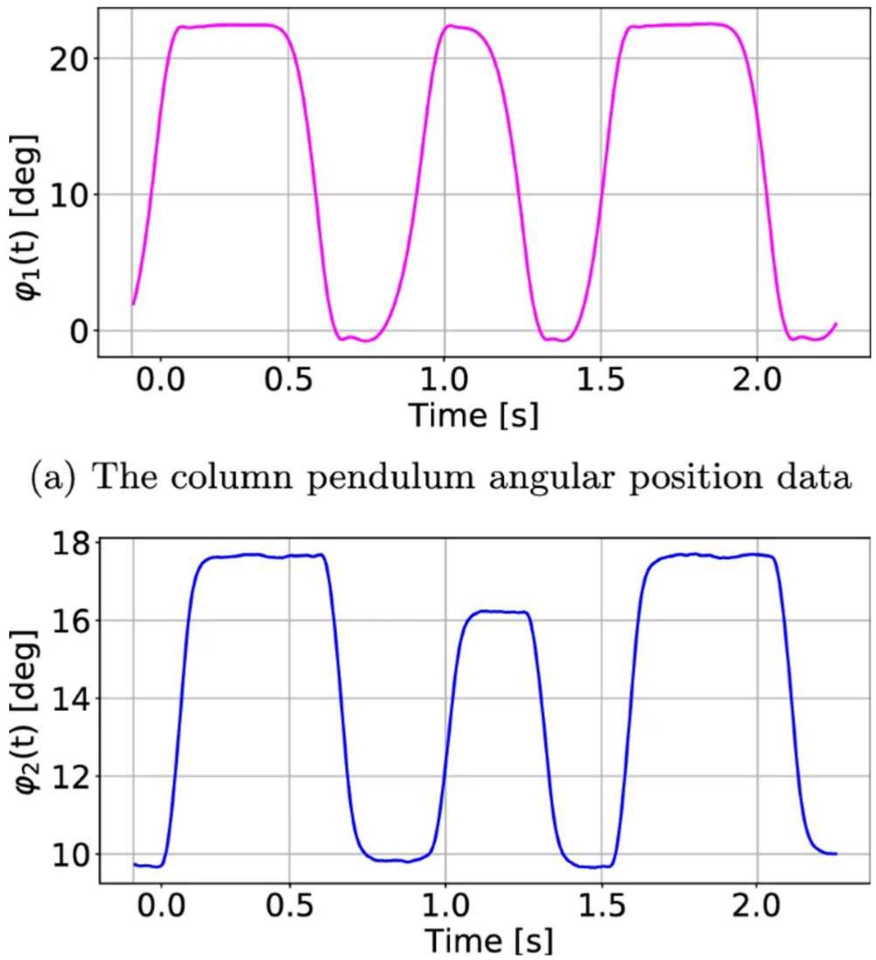


Fig. 6.

5.2. DATA ACQUISITION



(a) The column pendulum angular position data

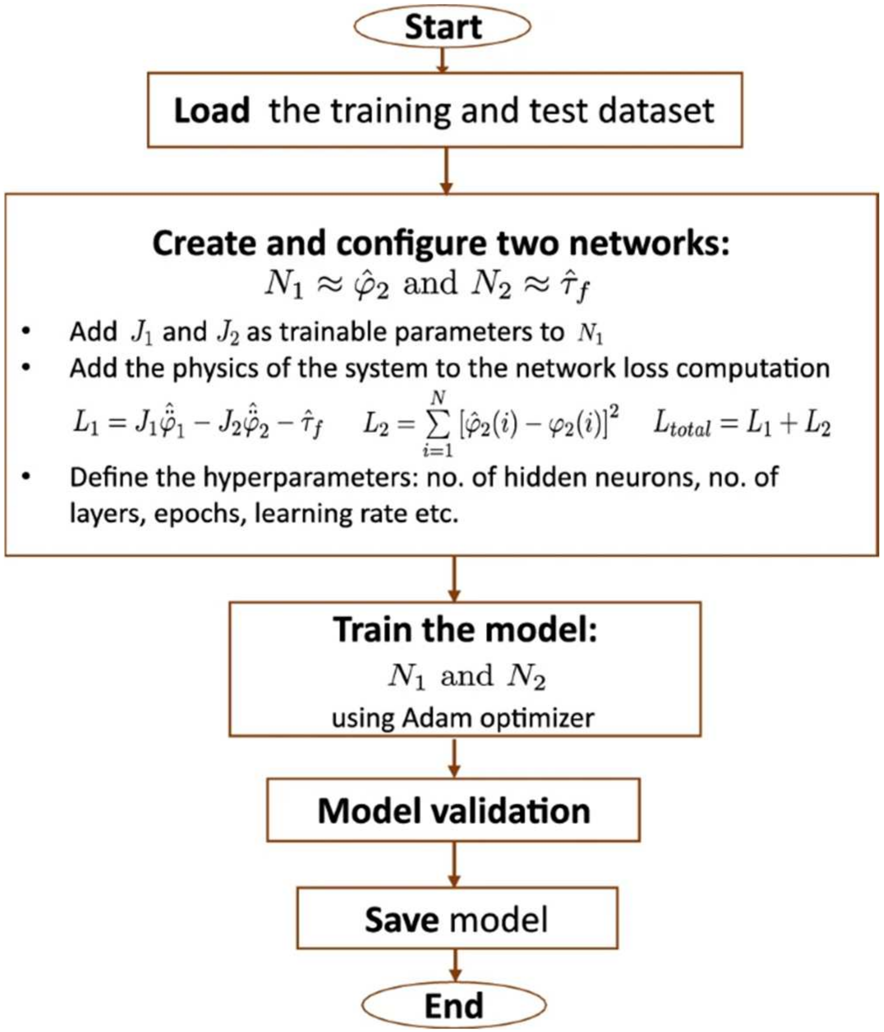
Fig. 7. (b) The disk pendulum angular position data

5.3 MODEL ESTIMATION USING NELDER–MEAD SIMPLEX DIRECT SEARCH ALGORITHM

After application of the Nelder–Mead simplex direct search algorithm, a prediction of the disk behavior shown in Fig. 8 a (gray line) and also a set of parameters of the friction model (6) is found: $T_{sl} = 0.2328$, $T_{sr} = 0.0928$ [N m], $T_{0l} = 0.2188$, $T_{0r} = 0.0917$ [s/rad], $\alpha = 92.6465$ [s/rad], $\beta = 0.0928$ [N m], $J_2 = 2.17 \cdot 10^{-4}$ [kg m^2]. The results of the real measurement series and numerical solutions demonstrate quite a good similarity between the mechanical system's response and its virtual analogue.

$$\begin{aligned} \tau_f(\dot{\varphi}_2) &= \frac{T_s}{1 + T_0\,|\dot{\varphi}_2|}\left(1 + \frac{\beta}{\cosh \alpha\dot{\varphi}_2}\right)\tanh \alpha\dot{\varphi}_2, \\ [T_s, T_0] &= \begin{cases} & \text{if} \quad \dot{\varphi}_2 \geq 0, \\ [T_{sr}, T_{0r}] & \text{if} \quad \dot{\varphi}_2 < 0, \end{cases} \end{aligned} \qquad (6)$$

5.4 MODEL ESTIMATION USING PINN ALGORITHM



5.4 MODEL ESTIMATION USING PINN ALGORITHM

Hyper parameter	Value
Number of hidden neurons	30
Hidden layer	2
Weight (w) and biases (b) initialization	Random
Activation function	a
Learning rate	0.01
Optimizer	ADAM
Number of epochs	10,000

[a]Tanh for hidden layers; pure linear for the output layer

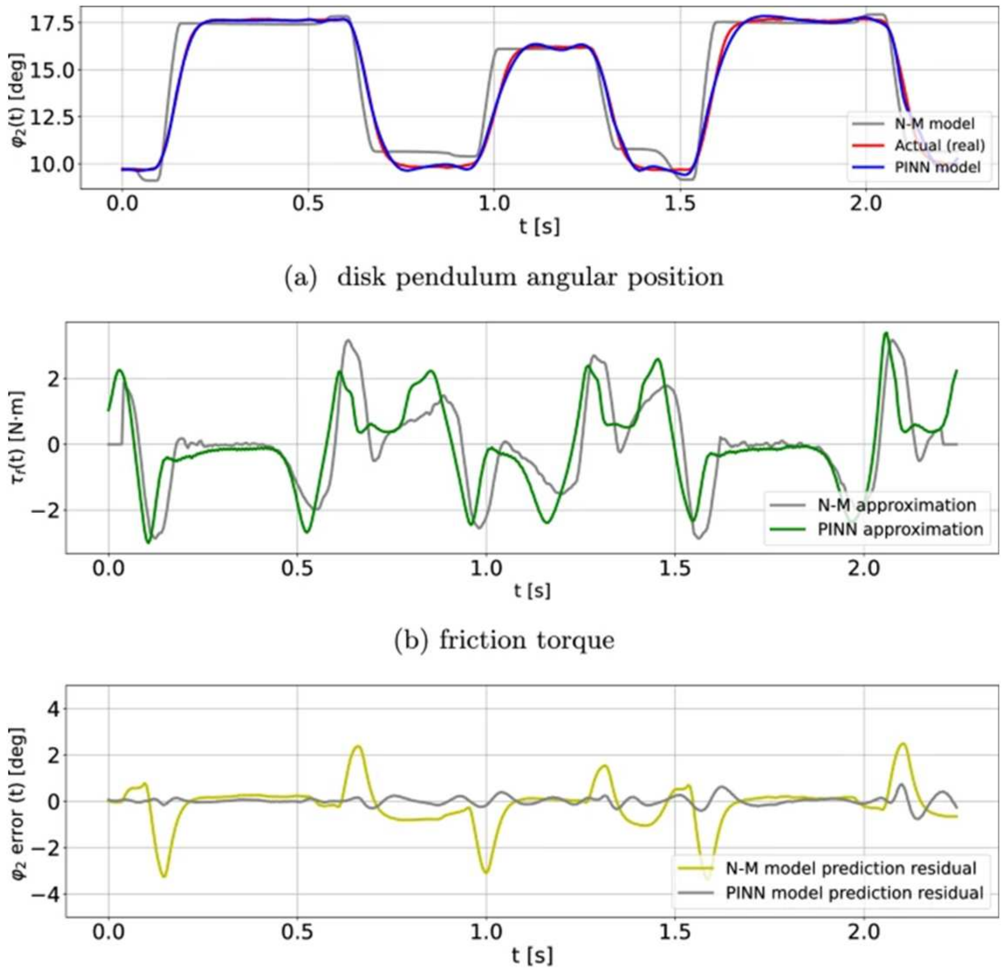
5.4 MODEL ESTIMATION USING PINN ALGORITHM

The loss function used in training N_1 is the mean squared error of the predicted disk pendulum angular rotation, while the loss function for N_2 is the computation of the residual of the system derived from Newton's second law of rotation:

$$J_2 \ddot{\varphi}_2 = \tau - \tau_f \; , \tag{7}$$

where $\tau = J_1 \ddot{\varphi}_1$ is the torque of the column pendulum, τ_f is the friction torque being sought, J_1 and J_2 are the mass moments of inertia of the column and disk pendulum, respectively.

6. RESULTS AND DISCUSSION



(a) disk pendulum angular position

(b) friction torque

(c) error of the model prediction

Fig. 8.

6. RESULTS AND DISCUSSION

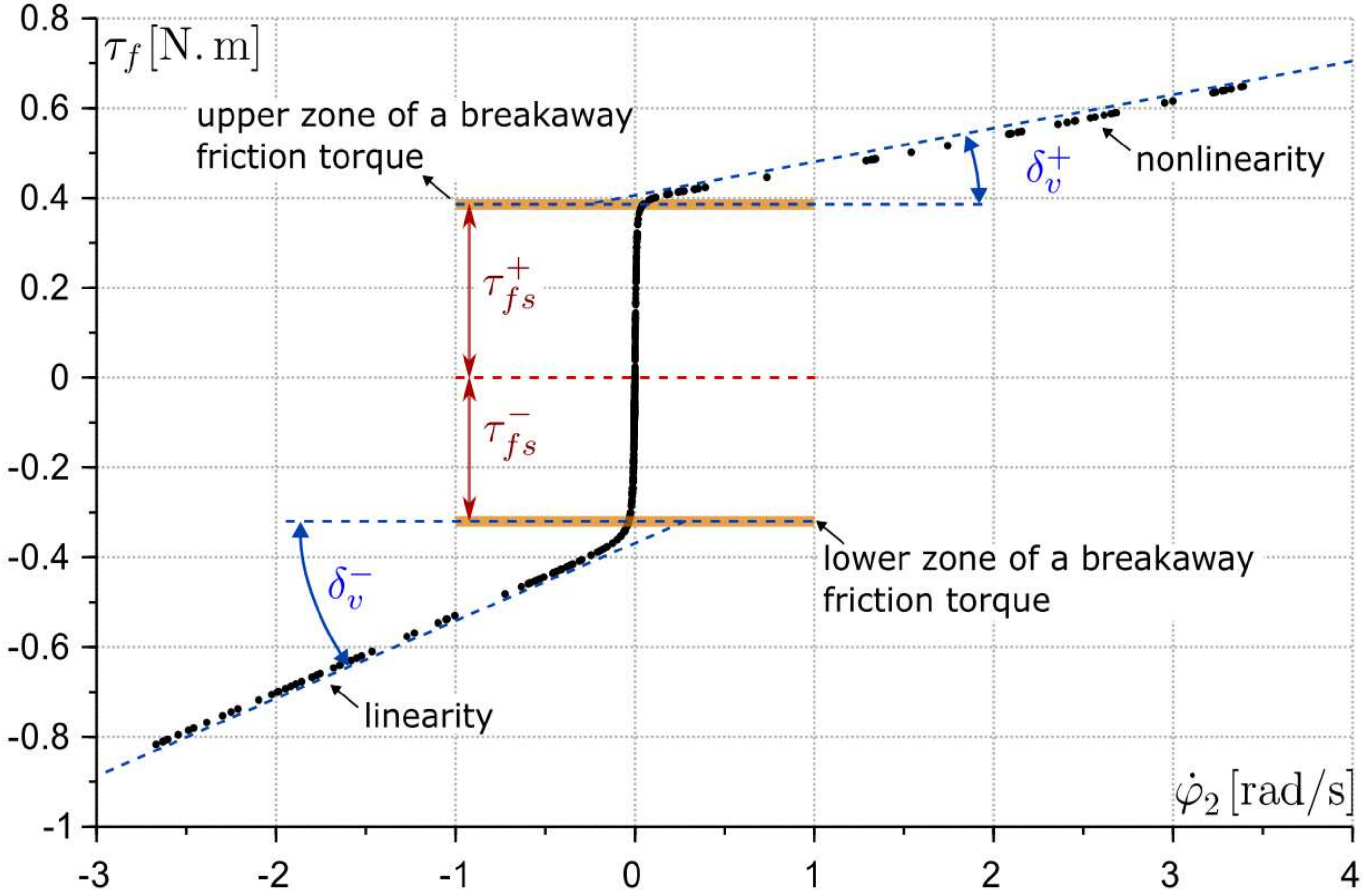


Fig. 9. The estimated non-symmetric friction torque characteristics

K11: 11. March 2025, 12:00, Room 2M334

CONCLUSIONS

A physics-informed neural network is suitable to predict the dynamic model and estimate planar friction law of a torsion pendulum system. The model was trained using time-series experimental data, and the results showed that the PINN model was able to accurately predict the angular rotation of the disk pendulum, while also estimating the planar friction between the pendulum bodies. The PINN model was able to identify the frictional loss in the system without using any pre-existing friction models and only relied on a simplified physics model with two estimated parameters. The approach based on the PINN algorithm proved to be faster and more accurate than the older Nelder–Mead method but requires further refinement due to the need for acquiring a broader knowledge of the friction model.

Thank You

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