Ulam Program

Chaotic and nonlinear vibrations in continuous rotating systems with AMB

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With sincere gratitude to **Prof. Awrejcewicz** for his valuable collaboration and guidance

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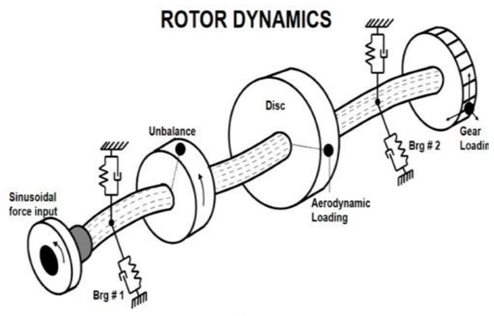
Understanding Rotor Dynamics

Key Concepts, Applications, Modeling, and Analysis



1. Rotor Dynamics

- Rotor dynamics studies vibrations in rotating machinery components such as shafts, turbines, and compressors.
- Vibrations can cause deformation, misalignment, and failure, making their analysis crucial.
- Applications include turbines, pumps, electric motors, and centrifuges.

2. Why is rotor dynamics critical?

Rotor dynamics deals with rotating machinery.

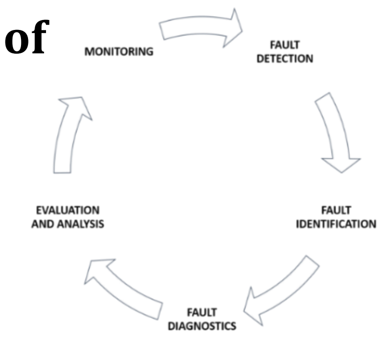
Reasons for the importance of this filed

- ### Application in Machine Design:

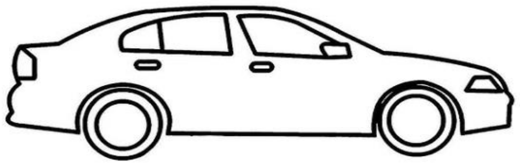
The rotating machinery design must ensure that vibrations remain within permissible limits at all operating speeds. This is particularly critical at high speeds, as excessive vibration can lead to **fatigue, rotor-stator contact, excessive bearing loads, and noise,...**

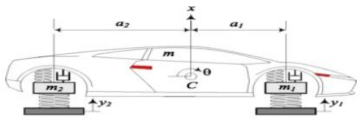
- ### Fault Diagnosis and Correction:

Its role in diagnosing and correcting a rotor's dynamic behavior. It enables the **identification of potential faults** and determining how to address them.



3. Steps to analyze any mechanical systems





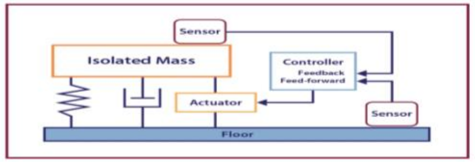
$$m\ddot{x} + c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) + k_1(x - x_1 - a_1\theta) + k_2(x - x_2 + a_2\theta) = 0$$

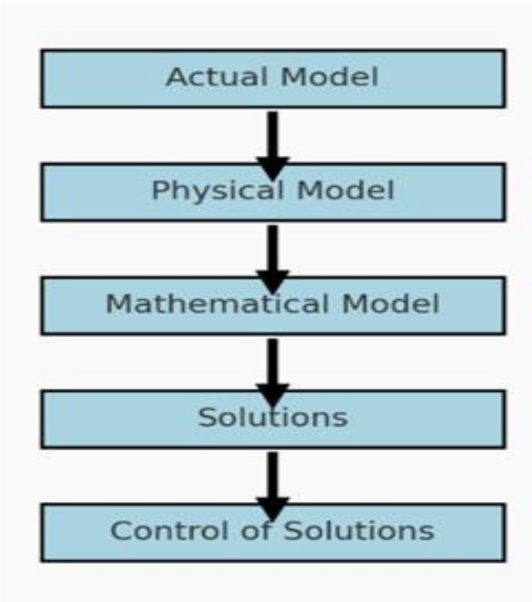
$$I_y\ddot{\theta} - a_1c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + a_2c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) - a_1k_1(x - x_1 - a_1\theta) + a_2k_2(x - x_2 + a_2\theta) = 0$$

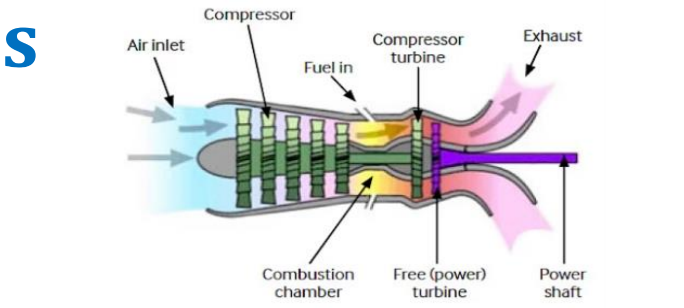
$$m_1\ddot{x}_1 - c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + k_{t_1}(x_1 - y_1) - k_1(x - x_1 - a_1\theta) = 0$$

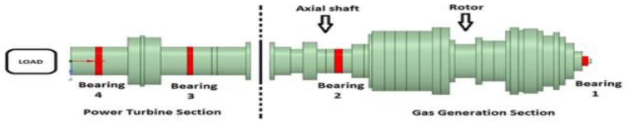
$$m_2\ddot{x}_2 - c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) + k_{t_2}(x_2 - y_2) - k_2(x - x_2 + a_2\theta) = 0$$

$$\mathbf{x} = \begin{bmatrix} x \\ \theta \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \sin\omega t + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \cos\omega t = \mathbf{A}\sin\omega t + \mathbf{B}\cos\omega t$$



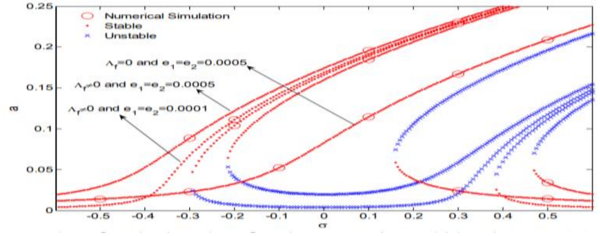


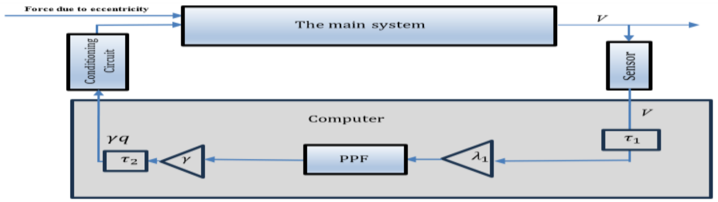




$$I_q\ddot{V} + c\dot{V} + 4\alpha n^4(V^3 + VW^2) + n^2 I_p\Omega_0\dot{W} + \varepsilon^2 n^2 I_p\Omega_0\dot{W}\cos(2\omega t) + \varepsilon^2 2n^2 I_p\Omega_0\omega W\sin(2\omega t) - n^2\Delta_I(4\dot{V}\Omega_0 - \ddot{W})\sin(2\Omega_0 t) + n^2\Delta_I(4\dot{W}\Omega_0 + \ddot{V})\cos(2\Omega_0 t) + n^4 V + n^4\Delta_D(V\cos(2\Omega_0 t) + W\sin(2\Omega_0 t)) = \Omega_0^2(e_1\cos\Omega_0 t - e_2\sin\Omega_0 t)$$

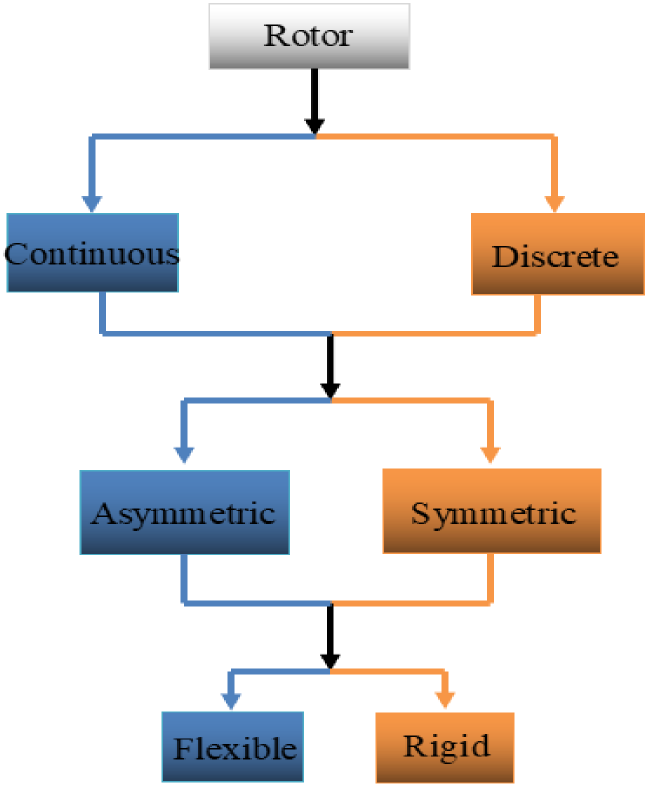
$$I_q\ddot{W} + c\dot{W} + 4\alpha n^4(W^3 + WV^2) - n^2 I_p\Omega_0\dot{V} - \varepsilon^2 n^2 I_p\Omega_0\dot{V}\cos(2\omega t) + n^2\Delta_I(\ddot{V} + 4\Omega_0\dot{W})\sin(2\Omega_0 t) + n^2\Delta_I(\ddot{W} - 4\dot{V}\Omega_0)\cos(2\Omega_0 T_0) + n^4 W + n^4\Delta_D(V\sin(2\Omega_0 t) - W\cos(2\Omega_0 t)) = \Omega_0^2(e_1\sin\Omega_0 t + e_2\cos\Omega_0 t)$$





4. Rotor Modeling

Various physical models for a rotor

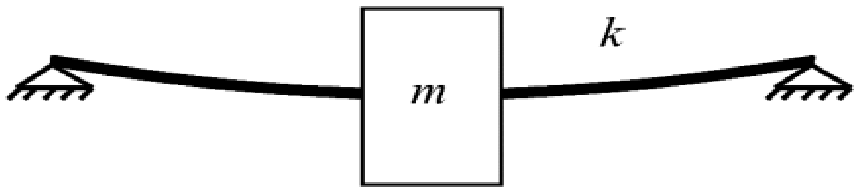


4.1. Lumped-mass Method/ Discrete method

- **Approach:** the rotor system is modeled discretely by representing its properties (mass, stiffness, and damping) as lumped parameters

- **Types of equations:** ODEs

- **Advantages:** Easier to set up and compute for smaller, less complex systems (rough estimation)



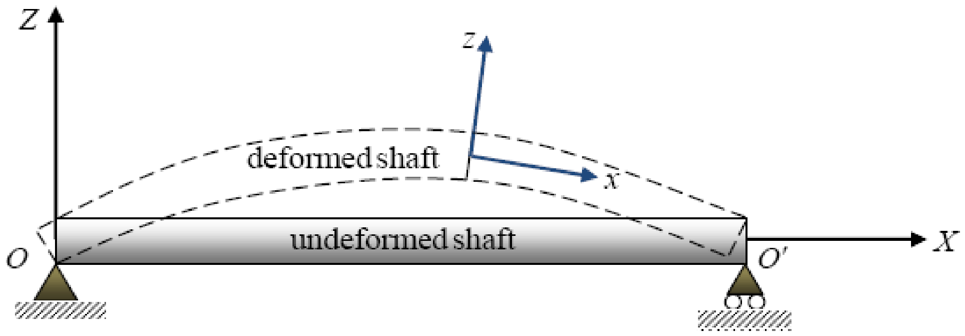
A flexible rotor with rigid supports (Jeffcott rotor/ lumped-mass model)

4.2. Continuous Method

- **Approach:** The rotor and its components are modeled as continuous entities, with mass, stiffness, and damping distributed along their length

- **Types of equations:** PDEs

- **Advantages:** Provides a highly detailed and accurate representation of dynamic behavior

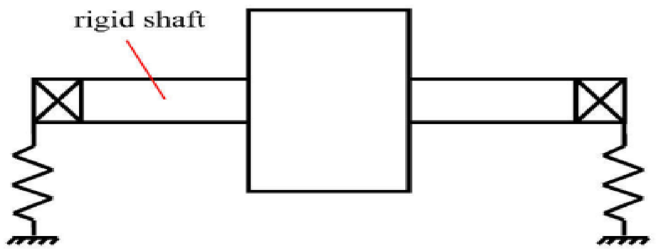


A continuous rotor

4.3. Rigid Rotor Method

- **Approach:** The rotor is modeled as a rigid body, assuming no deformation within the rotor itself, and the flexibility of the system is considered only in the supports (e.g., bearings)

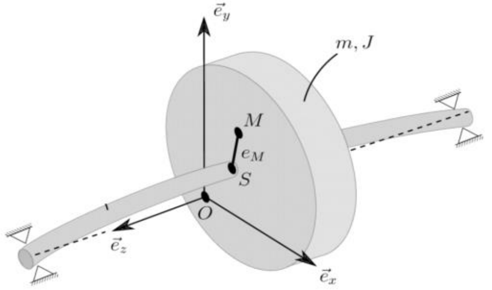
- **Advantages:** Simplifies calculations while still capturing the essential dynamics in specific scenarios



A Rigid rotor with flexible supports

4.4. Symmetrical and asymmetrical rotor

Symmetrical rotor: A symmetrical rotor is a type of rotor in mechanical or electrical systems that has uniform mass distribution and identical geometric properties around its axis of rotation



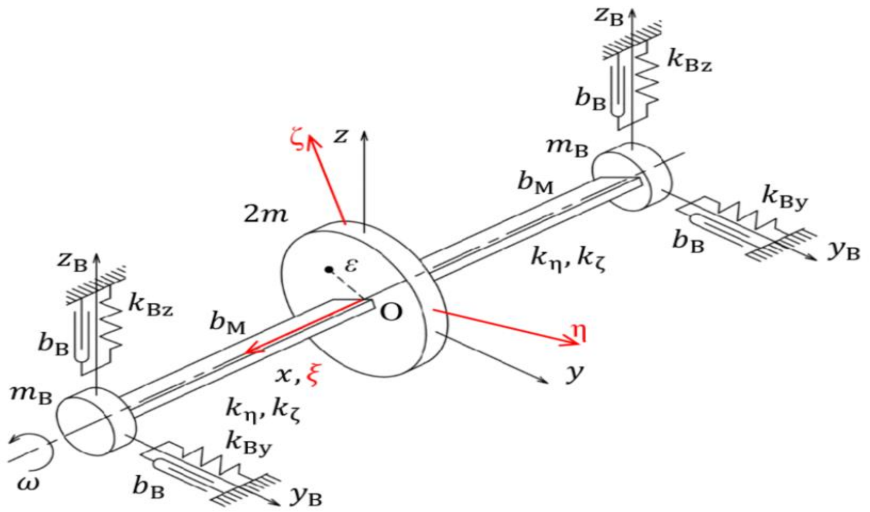
a typical symmetrical rotor

Asymmetrical rotor: An asymmetrical rotor is a type of rotor that does not have uniform mass distribution or identical geometric properties around its axis of rotation. (Following figure: a rotor of the two-pole alternating current generator with slots for coils, creating different bending stiffness in different planes)

Two-pole generator rotor

4.5. Rotor with anisotropy

- Anisotropy in rotors caused by bearings occurs when the support system (bearings) exhibits directionally dependent stiffness, damping, or other mechanical properties



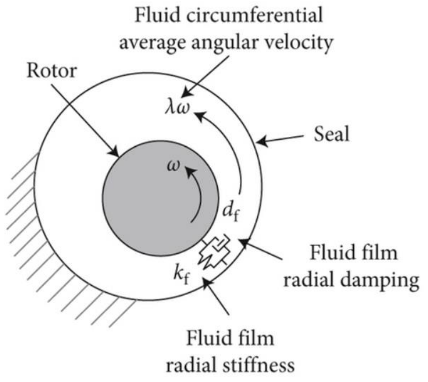
4.6. Linear and nonlinear models

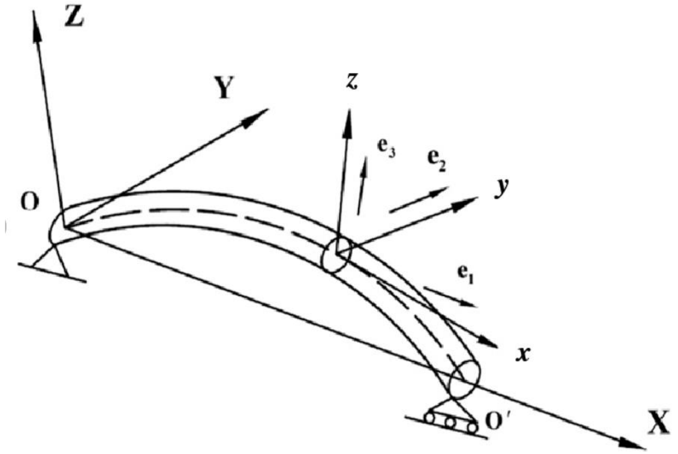
Aspect	Linear Rotor	Nonlinear Rotor
Motion	Small, proportional deflections	Large or moderately large, complex deflections
Material Behavior	Elastic only	Elastic, plastic, or nonlinear materials
Response	Proportional	This may include regular and irregular responses such as chaos, bifurcation, etc.

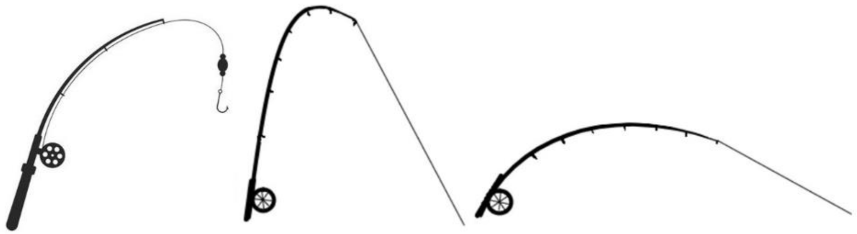
4.7. Linear and nonlinear models

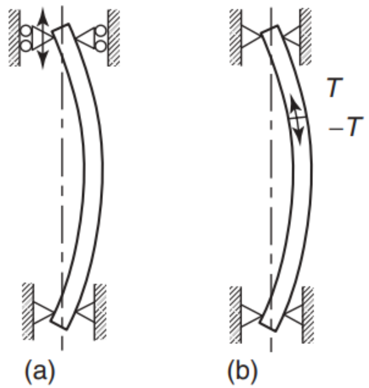
Sources of nonlinearities:

1. Inertia nonlinearity
2. Geometric nonlinearity
3. Fluid-induced
4. Rotor-stator contact

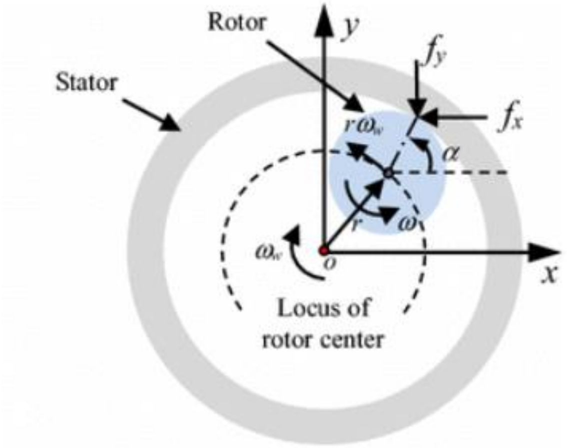






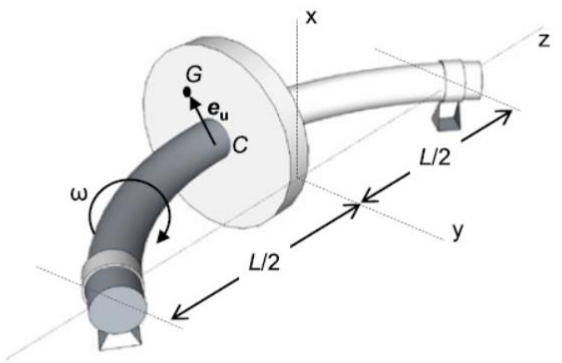


Geometrical nonlinearity: (a) linear system and (b) nonlinear system.

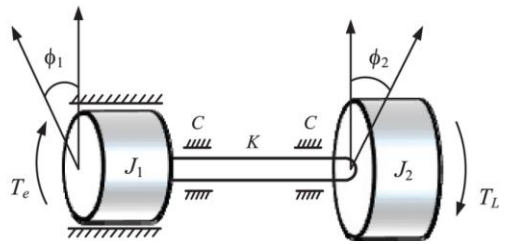


5. Different Types of Rotor Vibrations

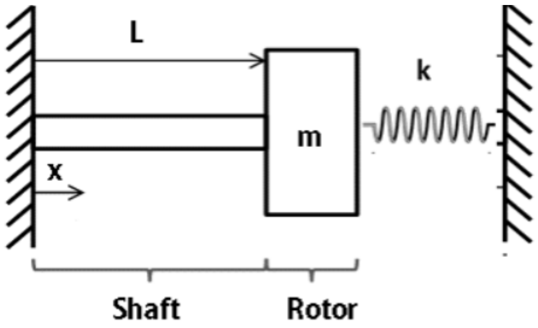
- Lateral Vibrations (Flexural/Transverse Vibrations)



- Torsional Vibrations



- Longitudinal Vibrations (Axial Vibrations)

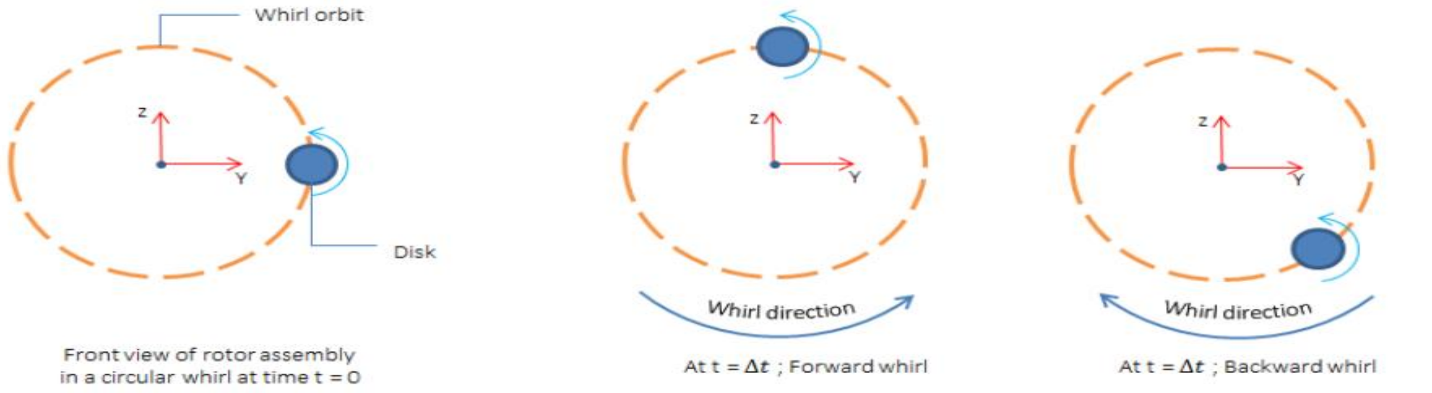


6. Some significant concepts in rotor dynamics:

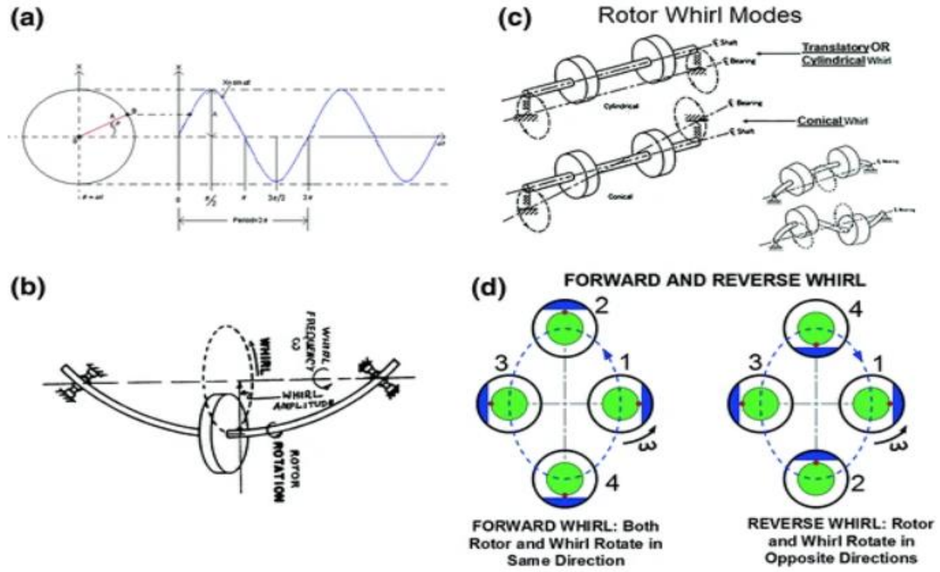
6.1. Forward and backward whirls:

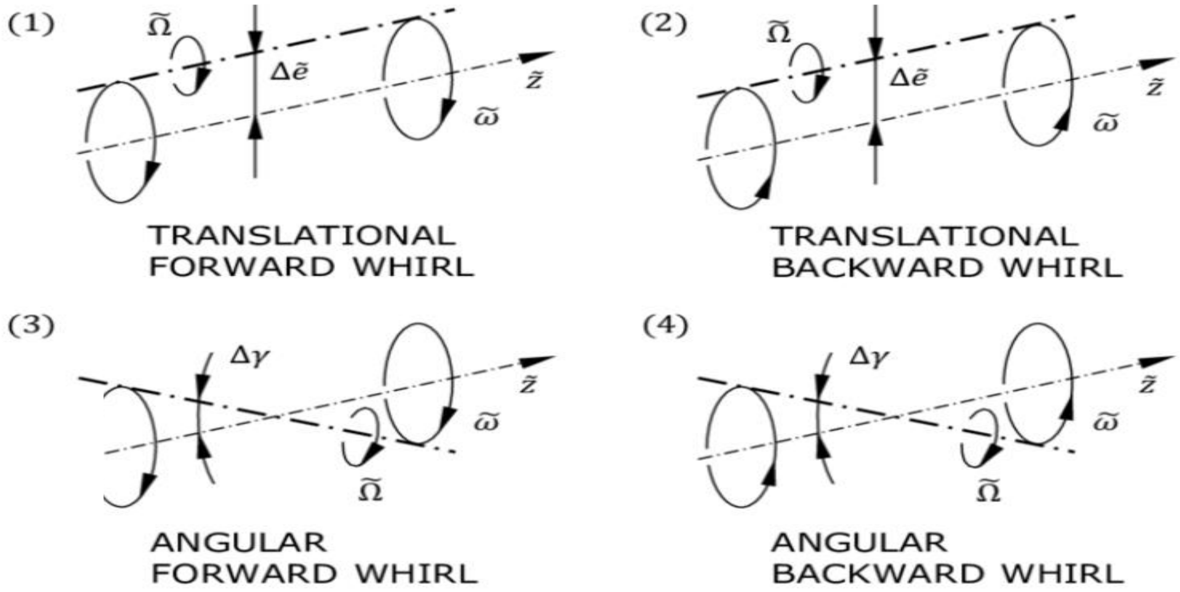
Whirl/whirling motion: whirl refers to the motion of a rotor's centerline as it moves away from the axis of rotation, typically forming a circular or elliptical path.

Different whirling motions: forward whirl and backward whirl based on the direction of the rotor's motion relative to its rotational direction.



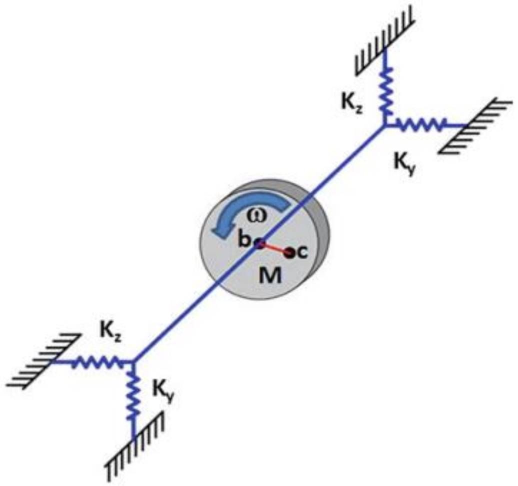
6.1. Forward and backward whirls

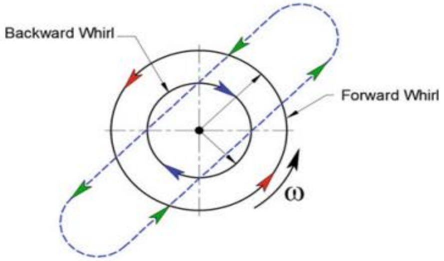


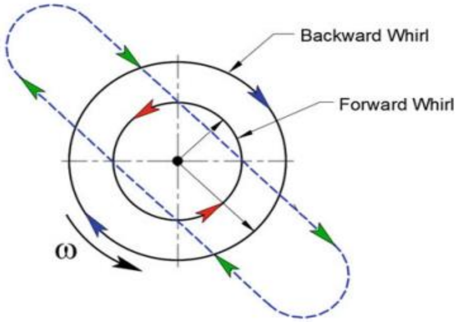


6.1. Forward and backward whirls

Any motion in a rotor can be expressed by backward and forward modes (whirling motions):







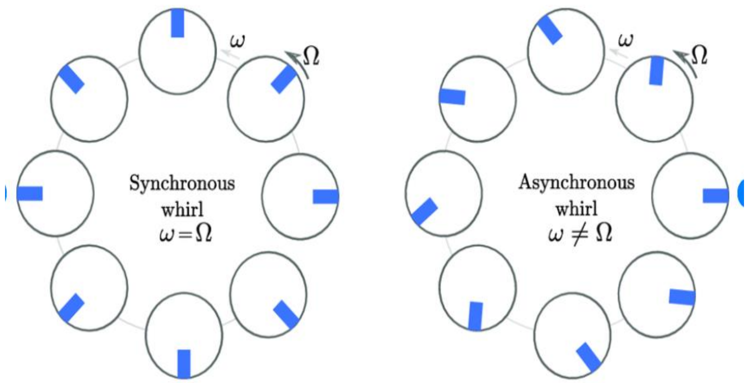
6.2. Synchronous and asynchronous Vibrations

6.2.1. Synchronous Vibrations

Vibrations that occur at the same frequency as the rotational speed of the rotor.

Practical example:

- Imagine a fan with a small weight attached to one blade. As the fan rotates, the imbalance causes the rotor to wobble at the same speed as its rotation.

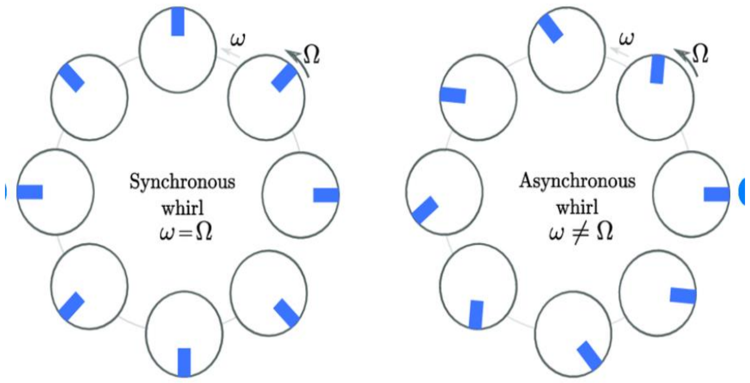


6.2.2. Asynchronous Vibrations

Vibrations that occur at a frequency different from the rotational speed of the rotor.

Practical example:

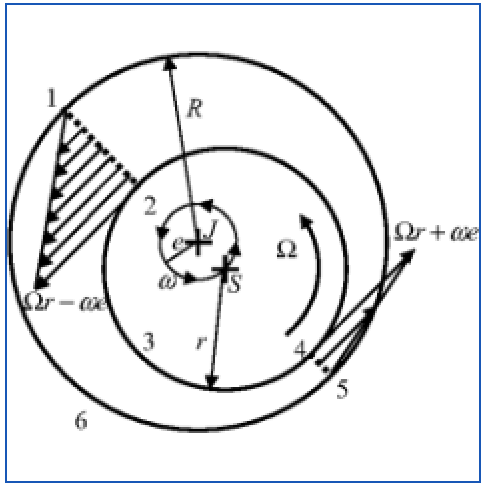
- Imagine the fan again, but this time, an interaction with air currents causes the vibration frequency to be slower or faster than the rotor's rotation speed.



6.3. oil whirl and oil whip

6.3.1. Oil Whirl:

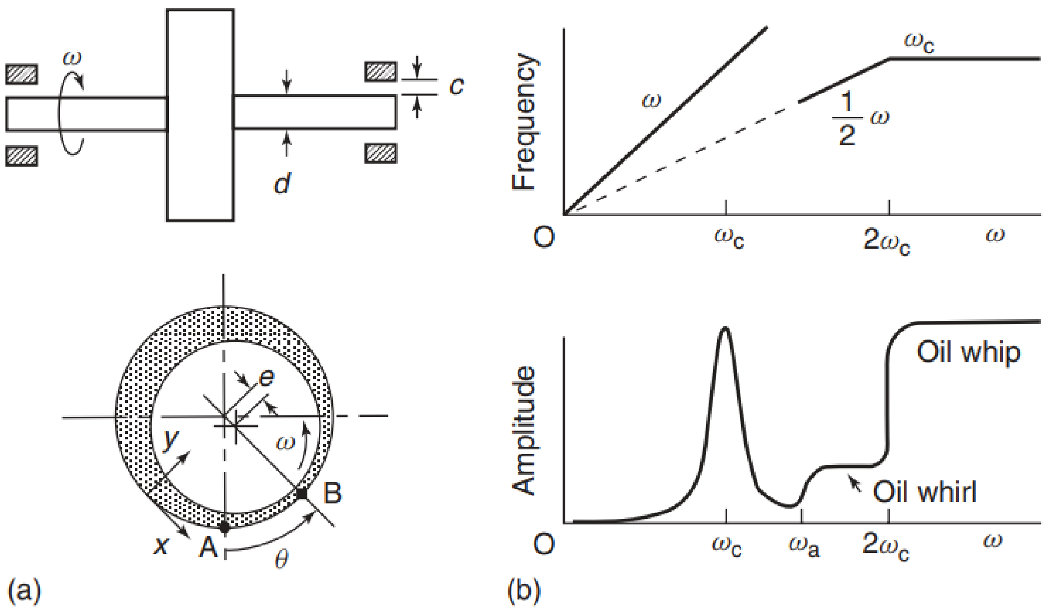
- Oil whirl occurs in rotating machinery, like turbines or compressors, with fluid-film bearings (bearings that use a thin layer of oil to support the shaft).
- **Sub-synchronous vibration:** As the shaft rotates, the oil inside the bearing can start to swirl or "whirl" around the shaft due to fluid dynamics. This creates a slight, repetitive vibration at a frequency slightly less than half the shaft's rotation speed (**around 0.4 to 0.48 times the shaft speed)**
- It's usually a minor issue unless it grows stronger or leads to instability.



6.3. oil whirl and oil whip

6.3.2. Oil Whip: Oil whip is a more severe version of oil whirl. It occurs when the vibration caused by the oil whirl grows so strong that it synchronizes with the natural frequency of the shaft.

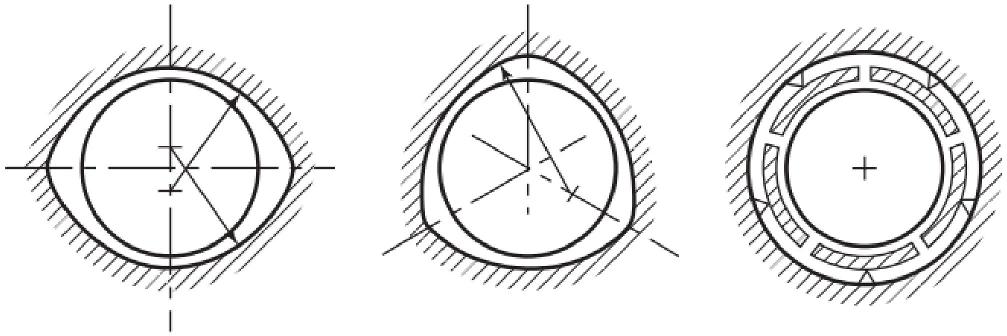
- The resonance amplifies the vibrations, causing the shaft to move erratically or "whip" around, potentially leading to damage or failure.
- Oil whip is dangerous and needs to be addressed immediately.



6.3. oil whirl and oil whip

Prevention of Oil Whip

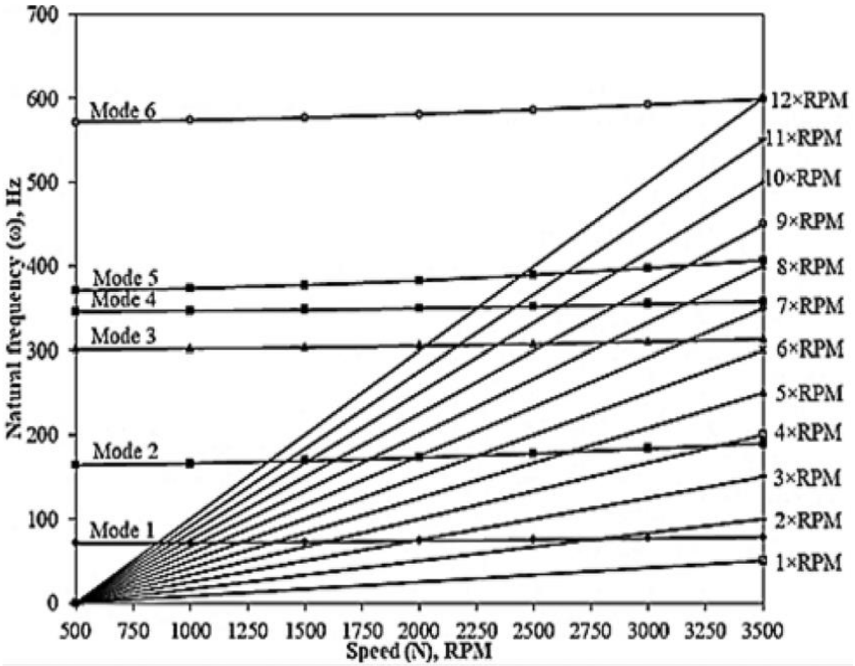
1. **Increase Natural Frequency**: By changing the rotor's design (e.g., shortening its length or increasing its diameter).
2. **Special Bearings**: Use bearings like tilting-pad or multi-lobe designs that are less likely to trigger oil whip.
3. **Increase Load or Clearance**: Adjusting these parameters can expand the range of stable operation.
4. **Change Fluid Properties**: Reducing viscosity or increasing eccentricity can help.

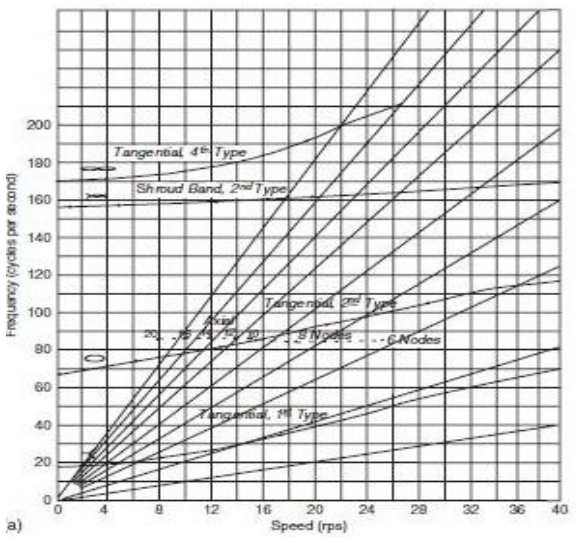


(a) (b) (c)

7. Campbell diagram

A Campbell Diagram is a graphical tool used in rotor dynamics to analyze the relationship between the natural frequencies of a rotating system and its rotational speed. It helps identify critical speeds and possible resonances.





8. Campbell diagram

Axes:

- X-axis: Rotational speed (usually in RPM or rad/s).
- Y-axis: Natural frequencies (usually in Hz or rad/s).

Lines:

- Natural frequency curves: These represent how the natural frequencies of the system change with rotational speed.
- Harmonic lines: These are straight lines representing multiples of the rotational speed (e.g., 1X, 2X, etc.).

Intersection Points:

- When a natural frequency curve intersects a harmonic line, it indicates a critical speed where resonance might occur.

9. Acquiring mathematical model from physical model

Key methods:

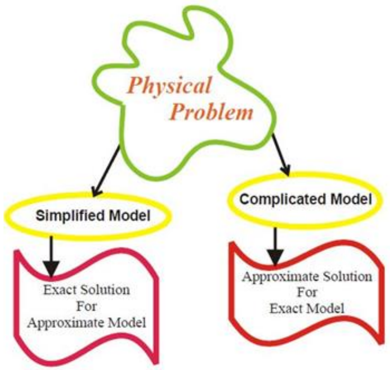
- Newton's Second Law
- Hamilton's Principle (Variational Principle)
- Lagrange's Equations of Motion
- Euler-Lagrange Equation (Variational Method)
- Energy Methods (Work-Energy Principle)
- Power balance method

Boris Galerkin (Russian)

9.1. Types of mathematical models

- ODE (Lumped-Mass Model)
- PDE (Continuous Model)

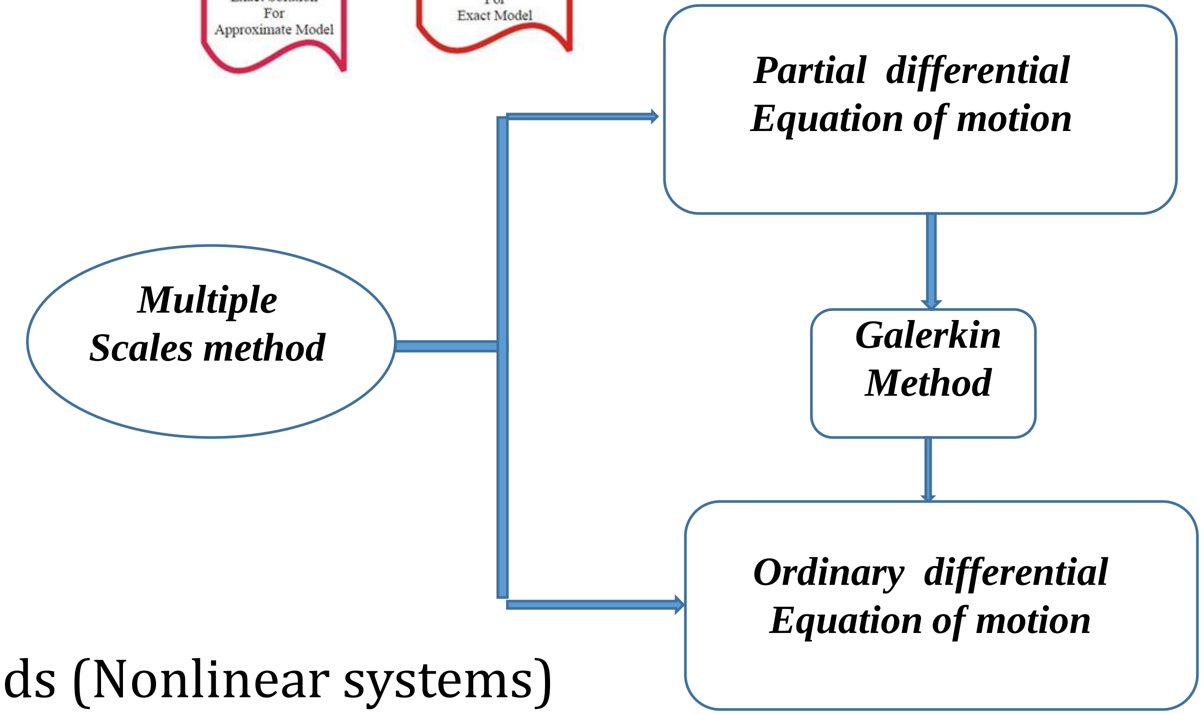
9.2. Solving Strategies



9.2.1. Solving PDEs (Partial Differential Equations)

Numerical Methods

- Finite Element Method (FEM):
- Finite Difference Method (FDM):
- Meshless Methods:
- Spectral Methods:



Analytical Methods

- Separation of Variables (Linear systems)
- weighted residual method/variational methods (Nonlinear systems)
- Transform Methods
- Perturbation Methods

9.2.2. Solving ODEs (Ordinary Differential Equations)

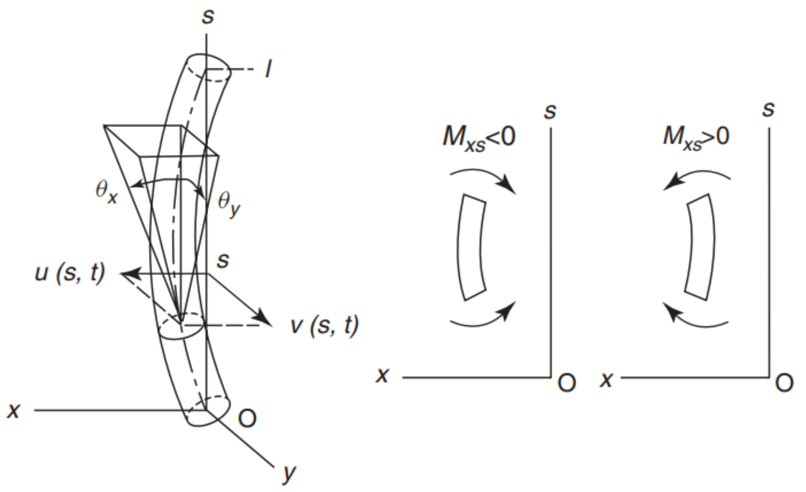
Numerical Methods

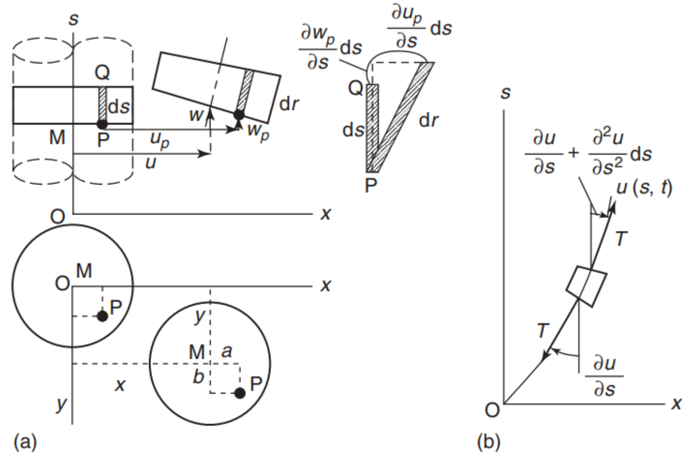
- Runge-Kutta Methods:
- Leapfrog Methods:
- Shooting Method:

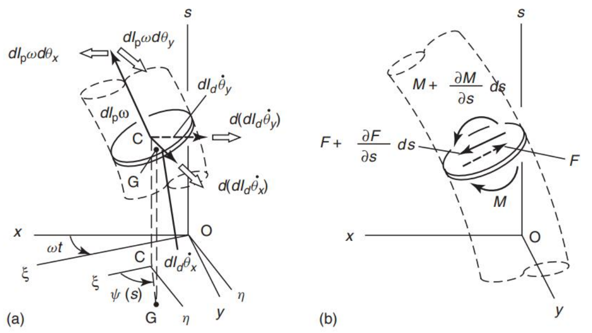
Analytical Methods

1. Perturbation Methods:
2. Homotopy Analysis Method:
3. Adomian Decomposition Method (ADM):

10. Capturing equations of motion for a continuous asymmetrical rotor using the Hamilton principle

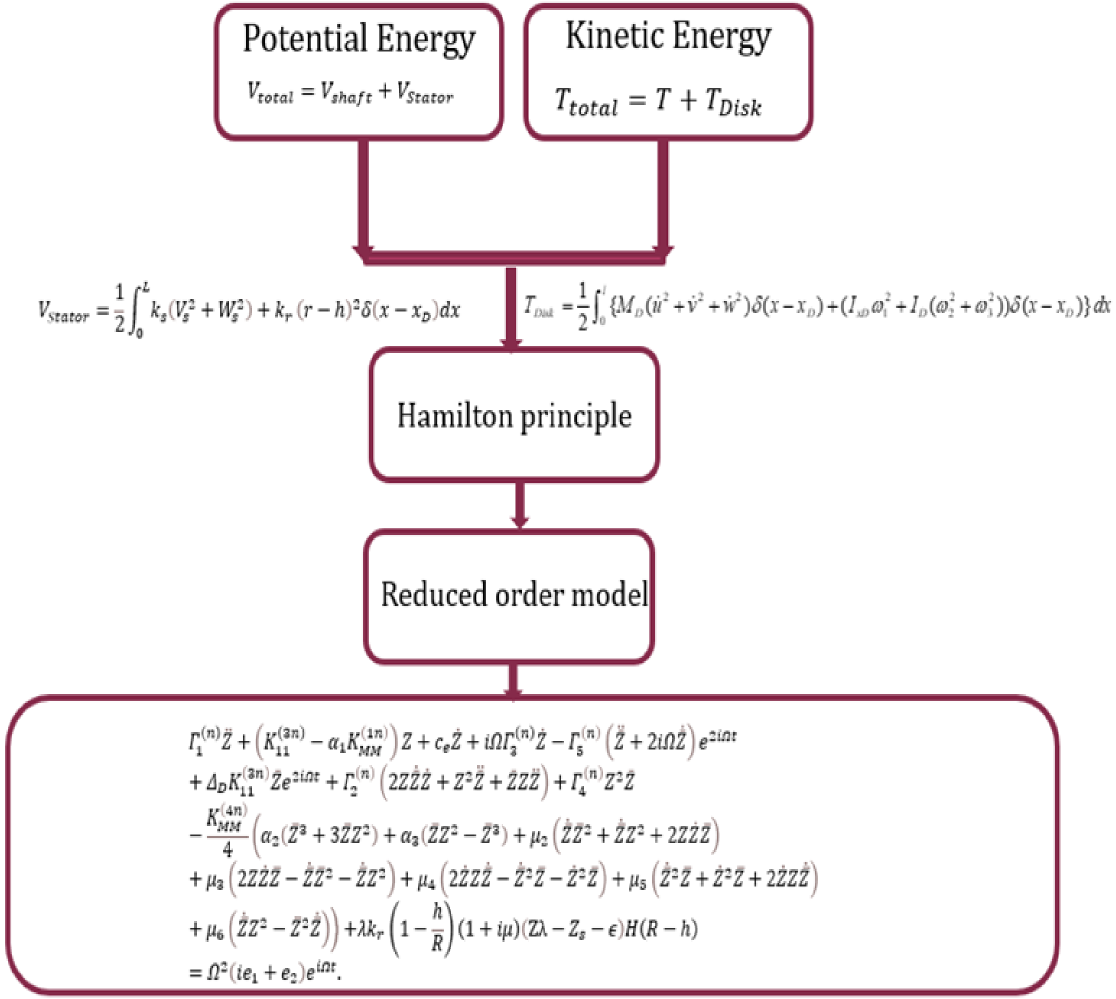






Free body diagrams for a continuous rotor

10. Capturing equations of motion for a continuous asymmetrical rotor using the Hamilton principle



11. Different nonlinear behaviors in an asymmetrical rotor:

11.1. Regular behaviors

- Primary resonance (due to imbalances and other forces)
- Secondary resonances (Sub-harmonic/ super-harmonic/ internal resonances)
- Combination resonances
- Parametric resonances (due to asymmetry)
- Hopf and double Hopf bifurcations

11.2. Irregular behaviors

- Chaos due to Rotor-stator rub impact
- Chaos arising from the destruction of homoclinic or heteroclinic loops (especially in AMB-rotor systems)

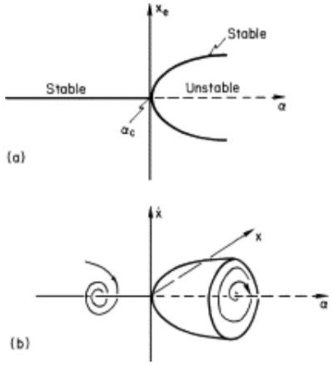
12. Regular behaviors in rotors

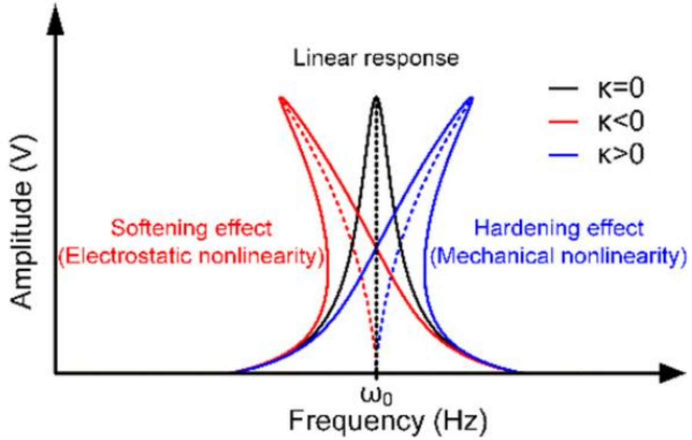
Goals (Bifurcations, stability, frequency response curves, the nature of responses: hardening/softening)

Summary of Nonlinear Resonances in a Continuous Rotor System.

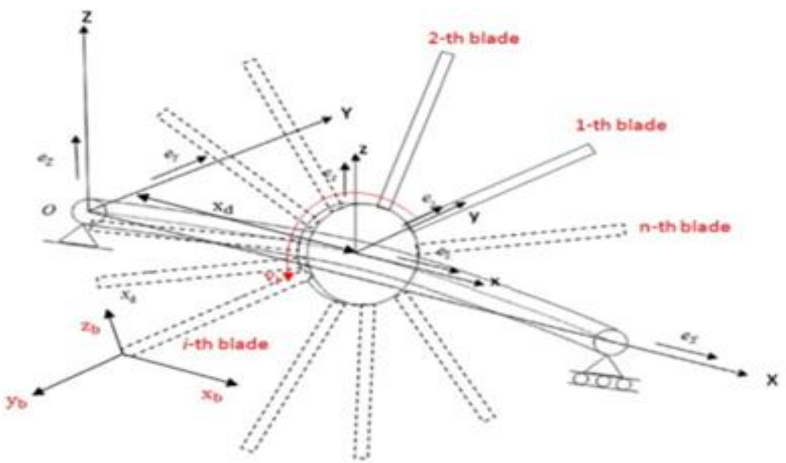
Type of resonance		**Linear**	**Nonlinear**
Main resonance	p_{fn}	Occurs	Occurs
	$2p_{fn}$		×
Subharmonic resonance	$-2p_{bn}$		×
	$3p_{fn}$		×
	$-3p_{bn}$		×
	$p_{fm} - p_{bn}$		×
	$p_{fm} + p_{fn}$		×
	$-p_{bm} - p_{bn}$		×
	$2p_{fm} + p_{fn}$		×
Combination resonance	$2p_{fm} - p_{bn}$		Occurs when $m = n$
	$p_{fm} - 2p_{bn}$		×
	$-2p_{bm} - p_{bn}$		×
	$p_{fl} + p_{fm} + p_{fn}$		×
	$p_{fl} + p_{fm} - p_{bn}$		Occurs when $l = n$ or $m = n$
	$p_{fl} - p_{bm} - p_{bn}$		×
	$-p_{bl} - p_{bm} - p_{bn}$		×

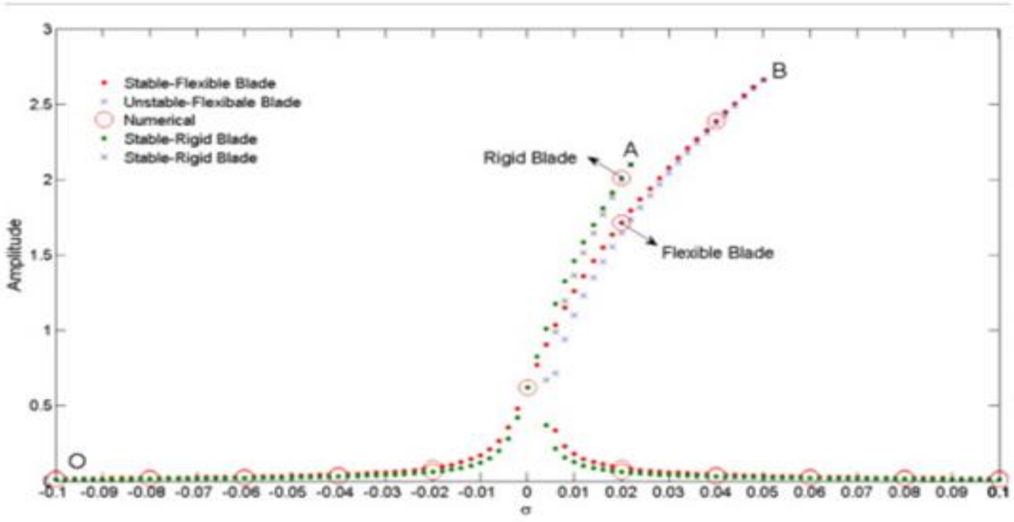
[a] The symbol × represents a case in which only the steady-state solution with zero-amplitude exists; in other words, this type of oscillation does not occur.





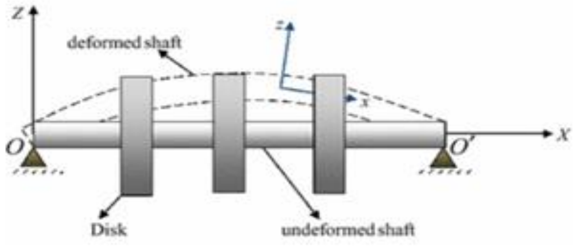
12.1. Vibration analysis of a symmetrical rotor with blades

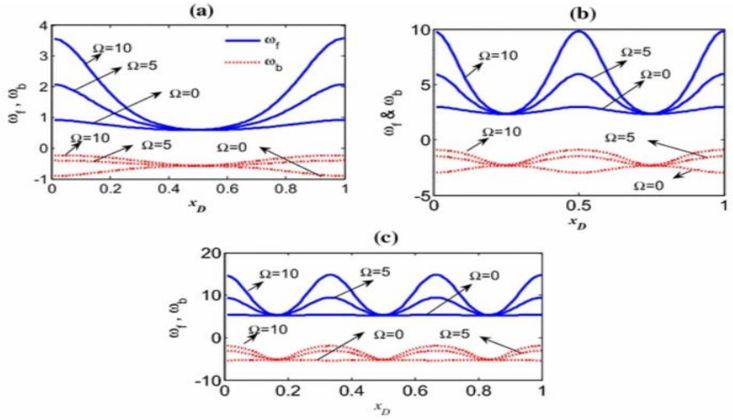


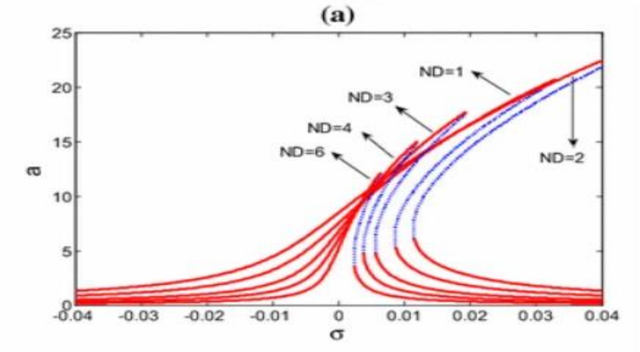


Frequency response *curves for a symmetrical* rotor with rigid/flexible blades

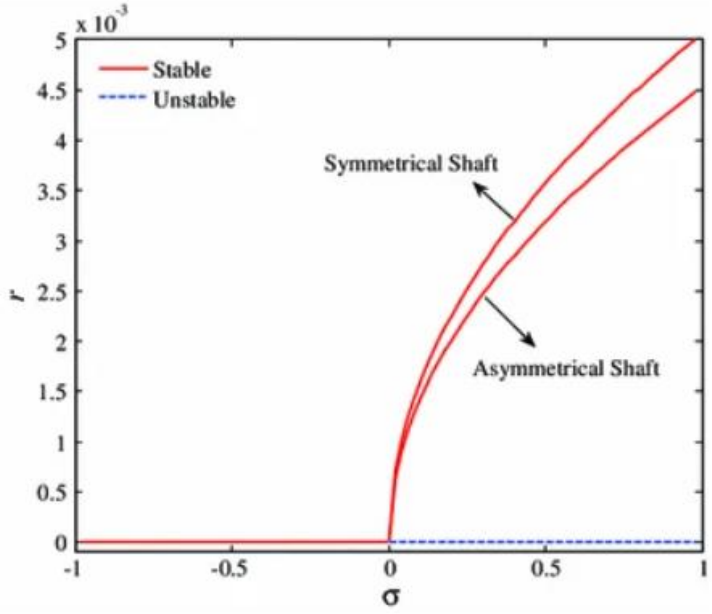
12.2. Vibration analysis of a multi-disk rotor

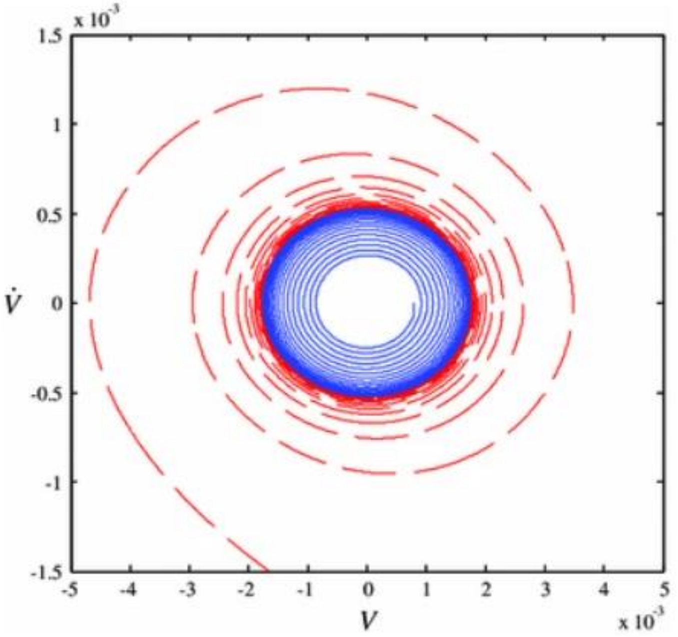




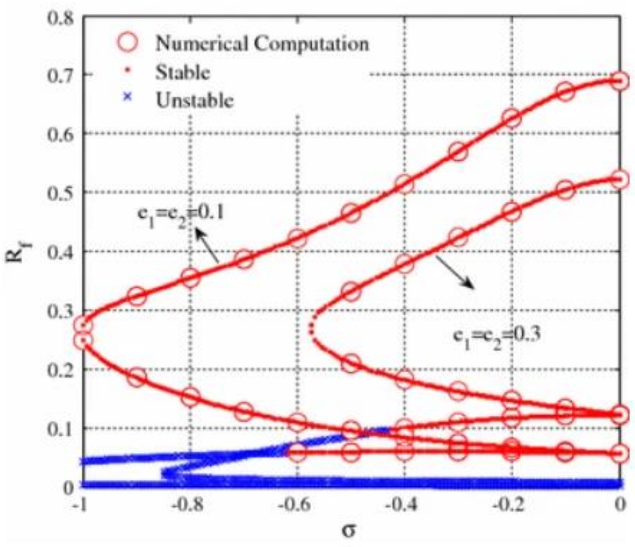


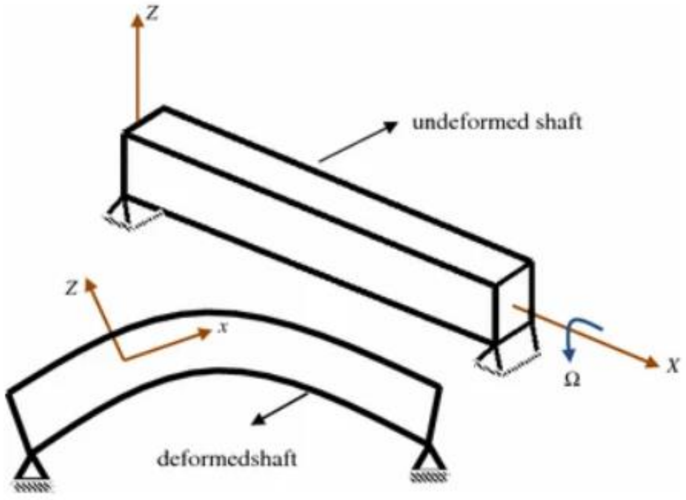
12.3. Hopf and double Hopf bifurcations analysis in an asymmetrical rotor



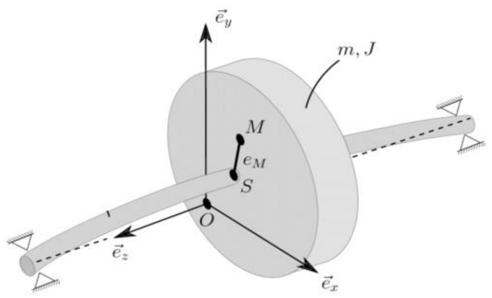


12.4. Internal, combinational, and sub-harmonic resonances of a nonlinear asymmetrical rotating shaft





13. Steady-sate responses of the symmetrical and asymmetrical rotors when the system operates near resonance



Steady-state responses

- **Symmetrical rotor**
  - Unbalanced system → Non-trivial solutions
  - Balanced system → trivial solutions
- **Asymmetrical rotor**
  - Unbalanced system → Non-trivial solutions
  - Balanced system → Non-trivial solutions; trivial solutions

14. Active Magnetic Bearings (ABM)

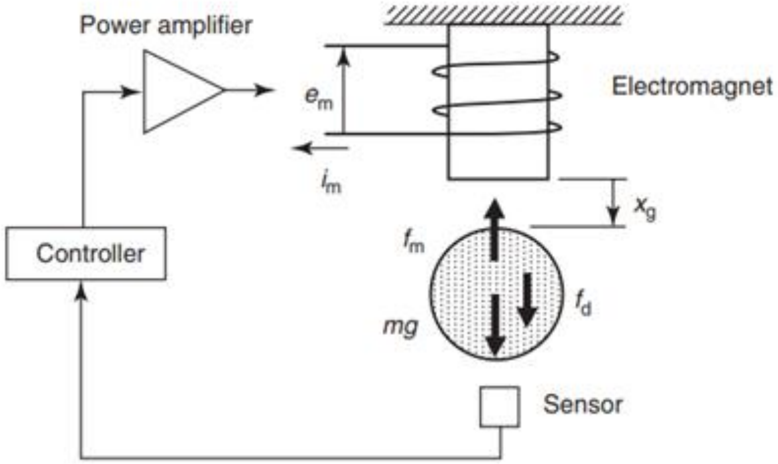
Contrary to journal and rolling bearings, which are associated with friction at the contact points, AMBs can support a rotor without contact.

14.1. Advantages of Active Magnetic Bearings (AMBs)

- Contact-Free Operation
- Low Friction
- Precise Control
- High Reliability (No lubrication required, simplifying maintenance)
- Low Maintenance (Significantly reduces maintenance frequency and costs)
- Wide Application Range (Ideal for high-speed and high-load applications such as Turbines, Compressors, and Aerospace systems)

15. Control mechanisms in AMBs:

the inherent characteristics of the AMBs produce negative stiffness, which can destabilize the system, so they need a controller.



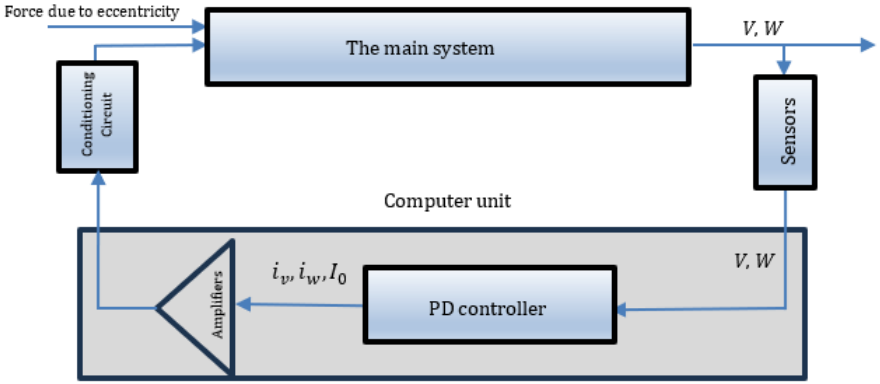
15.1. Some common control mechanisms:

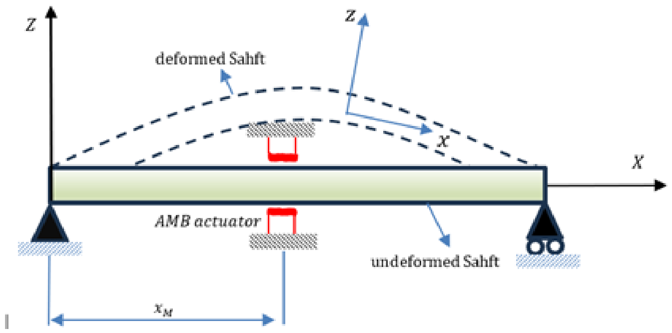
- Position-velocity controller (linear and nonlinear)
- NSC controller
- PPF controller
- APPF controller
- Velocity resonant feedback controller

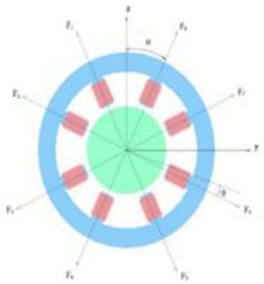
15.2. Applications of AMB:

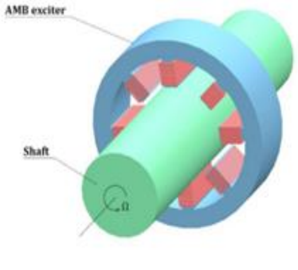
- As an actuator
- As bearings

16. An asymmetrical rotor with AMB

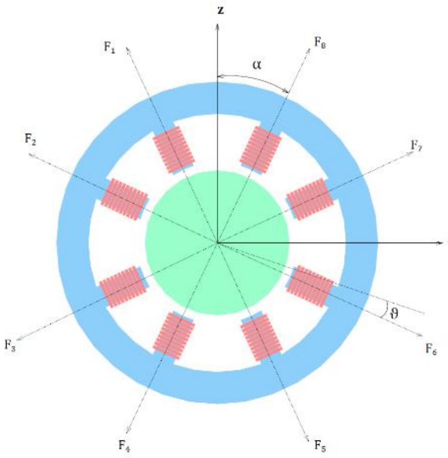




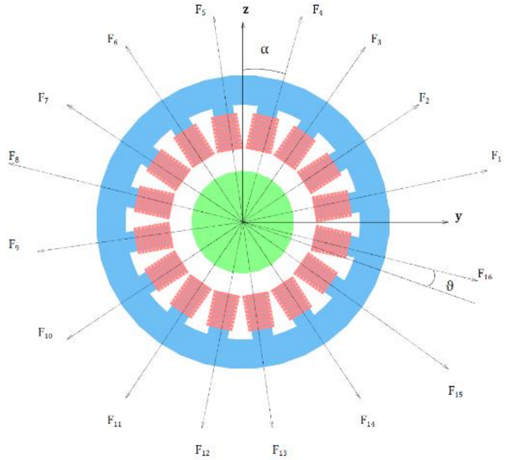




A rotor-AMB model



8-pole AMB



16-pole AMB

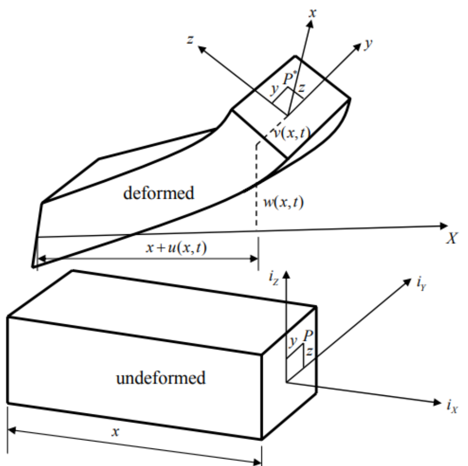
16. An asymmetrical rotor with AMB

$$T=\frac{1}{2}\int_0^l\Big(m(\dot u^2+\dot v^2+\dot w^2)+I_{xx}\omega_x^2+I_{yy}\omega_y^2+I_{zz}\omega_z^2+m\Omega^2\big[e_y^2(x)+e_z^2(x)\big]$$
$$-2m\Omega\big([e_z(x)\dot v+e_y(x)\dot w]\sin\beta+[e_y(x)\dot v-e_z(x)\dot w]\cos\beta\big)\Big)dx$$
$$+\frac{1}{2}m_s(\dot V_s^2+\dot W_s^2),$$

$$\Pi=\frac{1}{2}\int_0^l(N_{xx}e^2+D_{xx}k_x^2+D_{yy}k_y^2+D_{zz}k_z^2)dx,$$

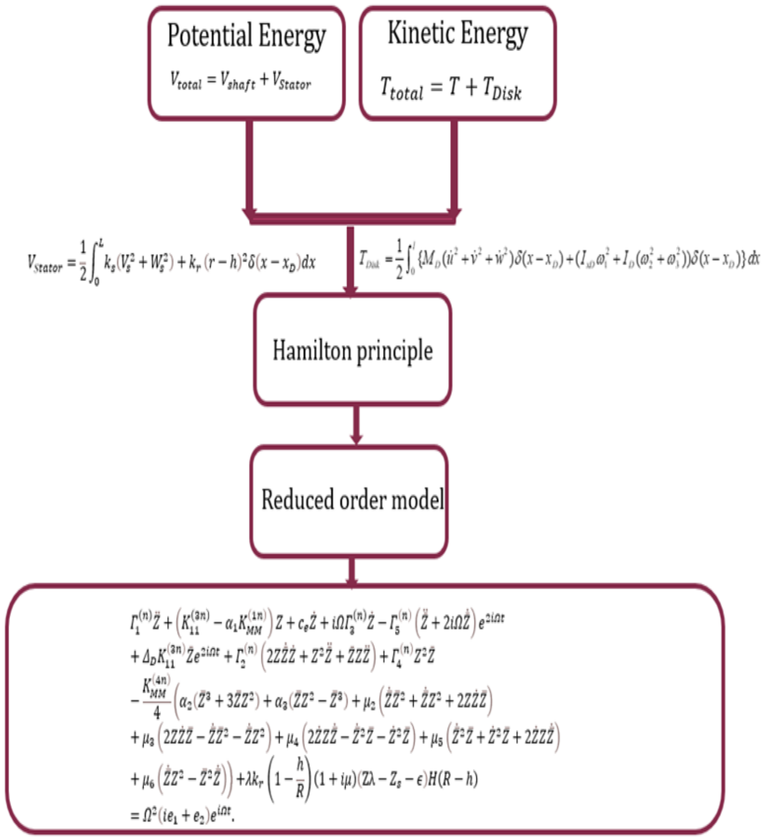
$$\Pi_{Stator}=\frac{1}{2}\int_0^L k_s(V_s^2+W_s^2)+k_r\,(r-h)^2\delta(x-x_D)dx,$$

$$\delta W_{ex}=Re\left(\int_0^l\big(c\dot z+F^M\delta(x-x_M)\big)\delta\bar z\,dx\right.$$
$$\left.+\,i\mu(z-Z_s-\epsilon)\delta(x-x_D)k_r\left(1-\frac{h}{r}\right)(\delta\bar z+\delta\bar Z_s)dx+c_sZ_s\delta\bar Z_s\right),$$

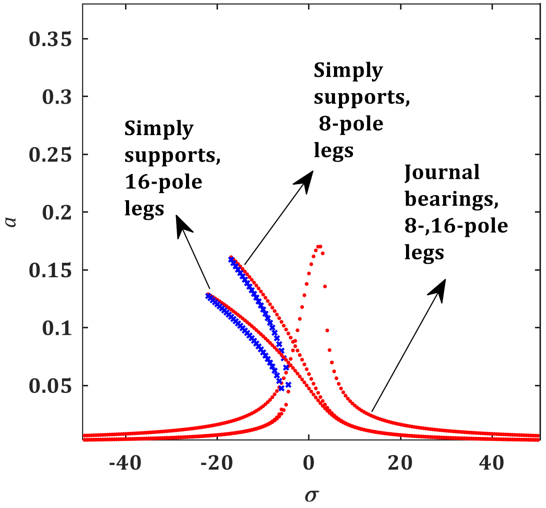


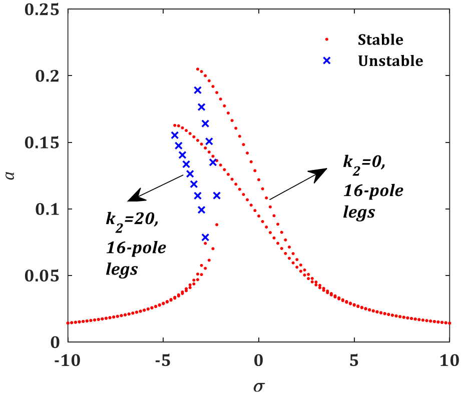
$$\Gamma_1^{(n)}\ddot Z+\left(K_{11}^{(3n)}-\alpha_1K_{MM}^{(1n)}\right)Z+c_e\dot Z+i\Omega\Gamma_3^{(n)}\dot Z-\Gamma_5^{(n)}\left(\ddot{\bar Z}+2i\Omega\dot{\bar Z}\right)e^{2i\Omega t}+\Delta_DK_{11}^{(3n)}\bar Ze^{2i\Omega t}$$
$$+\Gamma_2^{(n)}\left(2Z\dot{\bar Z}\dot Z+Z^2\ddot{\bar Z}+\bar ZZ\ddot Z\right)+\Gamma_4^{(n)}Z^2\bar Z$$
$$-\frac{K_{MM}^{(4n)}}{4}\Big(\alpha_2(\bar Z^3+3\bar ZZ^2)+\alpha_3(\bar ZZ^2-\bar Z^3)+\mu_2\left(\dot{\bar Z}\bar Z^2+\dot{\bar Z}Z^2+2Z\dot Z\bar Z\right)$$
$$+\mu_3\left(2Z\dot Z\bar Z-\dot{\bar Z}\bar Z^2-\dot{\bar Z}Z^2\right)+\mu_4\left(2\dot ZZ\dot{\bar Z}-\dot{\bar Z}^2\bar Z-\dot Z^2\bar Z\right)$$
$$+\mu_5\left(\dot{\bar Z}^2\bar Z+\dot Z^2\bar Z+2\dot ZZ\dot{\bar Z}\right)+\mu_6\left(\dot{\bar Z}Z^2-\bar Z^2\dot{\bar Z}\right)+\mu_7\left(\dot{\bar Z}^3+3\dot{\bar Z}\dot Z^2\right)\Big)+\lambda k_r(1$$
$$-\frac{h}{R})(1+i\mu)(Z\lambda-Z_s-\epsilon)H(R-h)=\Omega^2(ie_1+e_2)e^{i\Omega t}.$$

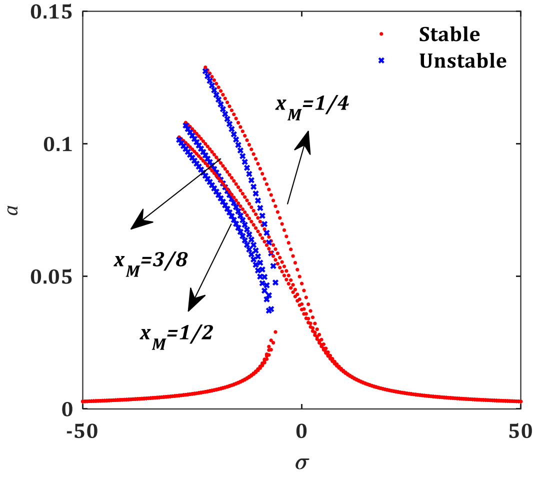
$$\ddot Z_s+k_sZ_s+c_s\dot Z_s=\beta\lambda k_r(1-\frac{h}{R})(1+i\mu)(Z\lambda-Z_s-\epsilon)H(R-h)$$

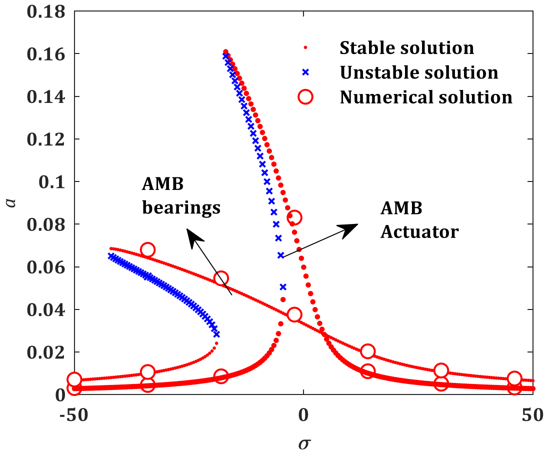


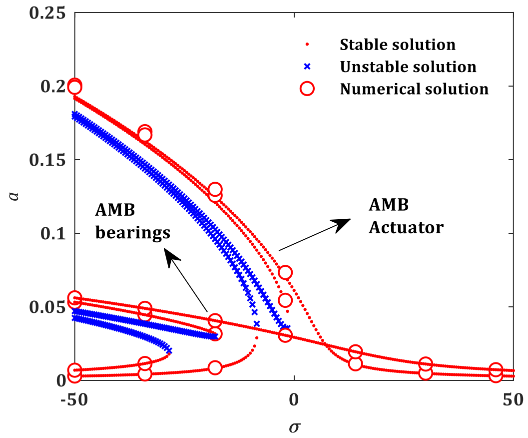
16. An asymmetrical rotor with AMB



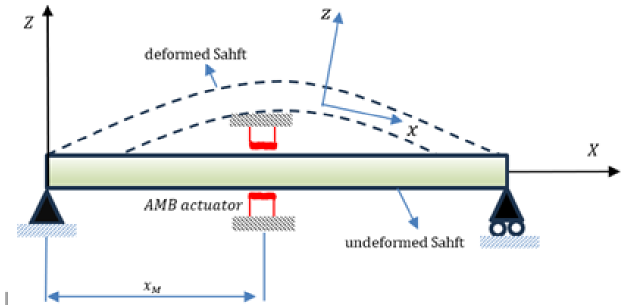






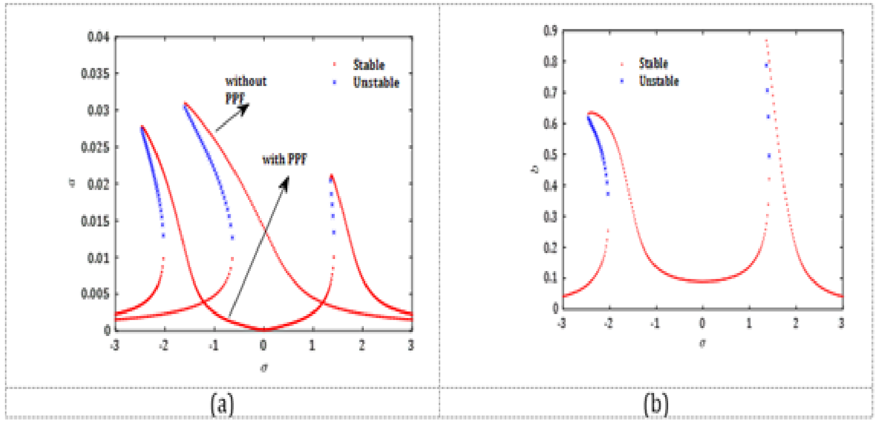


16.1.Nonlinear vibration control using PPF

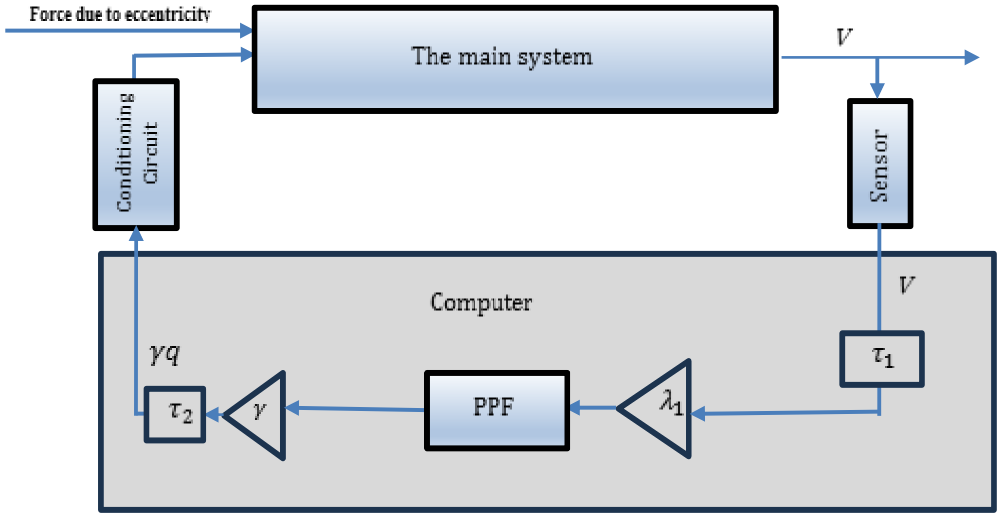


$$\Gamma_1^{(n)}\ddot{Z} + \left(K_{11}^{(3n)} - \alpha_1 K_{MM}^{(1n)}\right)Z + c_e\dot{Z} + i\Omega\Gamma_3^{(n)}\dot{Z} - \Gamma_5^{(n)}\left(\ddot{\bar{Z}} + 2i\Omega\dot{\bar{Z}}\right)e^{2i\Omega t} + \Delta_D K_{11}^{(3n)}\bar{Z}e^{2i\Omega t}$$
$$+ \Gamma_2^{(n)}\left(2Z\dot{Z}\dot{\bar{Z}} + Z^2\ddot{\bar{Z}} + \bar{Z}Z\ddot{Z}\right) + \Gamma_4^{(n)}Z^2\bar{Z}$$
$$- \frac{K_{MM}^{(4n)}}{4}\Big(\alpha_2(\bar{Z}^3 + 3\bar{Z}Z^2) + \alpha_3(\bar{Z}Z^2 - \bar{Z}^3) + \mu_2\left(\dot{\bar{Z}}\bar{Z}^2 + \dot{\bar{Z}}Z^2 + 2Z\dot{Z}\bar{Z}\right)$$
$$+ \mu_3\left(2Z\dot{Z}\bar{Z} - \dot{\bar{Z}}\bar{Z}^2 - \dot{\bar{Z}}Z^2\right) + \mu_4\left(2\dot{Z}Z\dot{\bar{Z}} - \dot{\bar{Z}}^2\bar{Z} - \dot{Z}^2\bar{Z}\right)$$
$$+ \mu_5\left(\dot{\bar{Z}}^2\bar{Z} + \dot{Z}^2\bar{Z} + 2\dot{Z}Z\dot{\bar{Z}}\right) + \mu_6\left(\dot{\bar{Z}}Z^2 - \bar{Z}^2\dot{\bar{Z}}\right) + \gamma q(t - \tau_2)\Big)$$
$$= \Omega^2(ie_1 + e_2)e^{i\Omega t}.$$

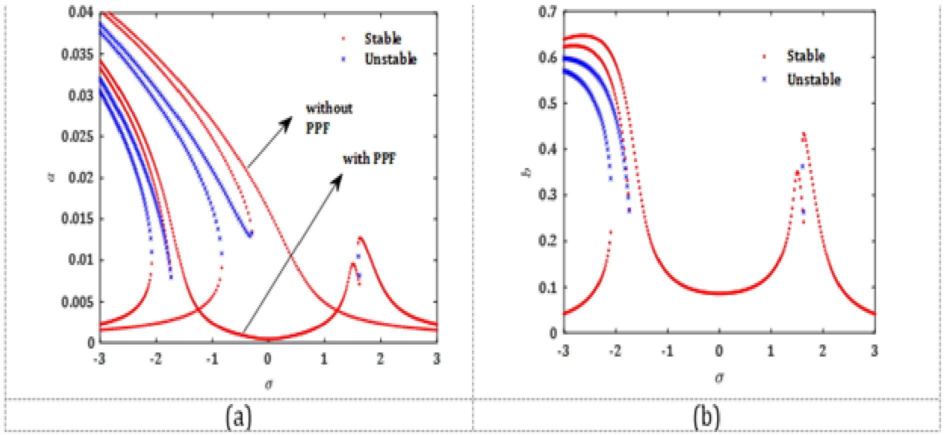
$$\ddot{q} + 2c_1\omega_1\dot{q} + \omega_1^2 q + \alpha_{cont}q^3 = \lambda_1 V(t - \tau_1),$$



Frequency response curves for symmetrical AMB-rotor system with PPF

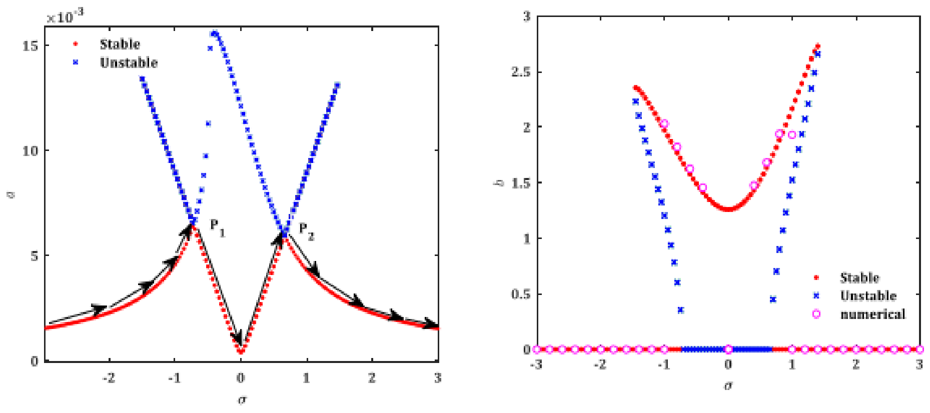


A PPF control mechanism



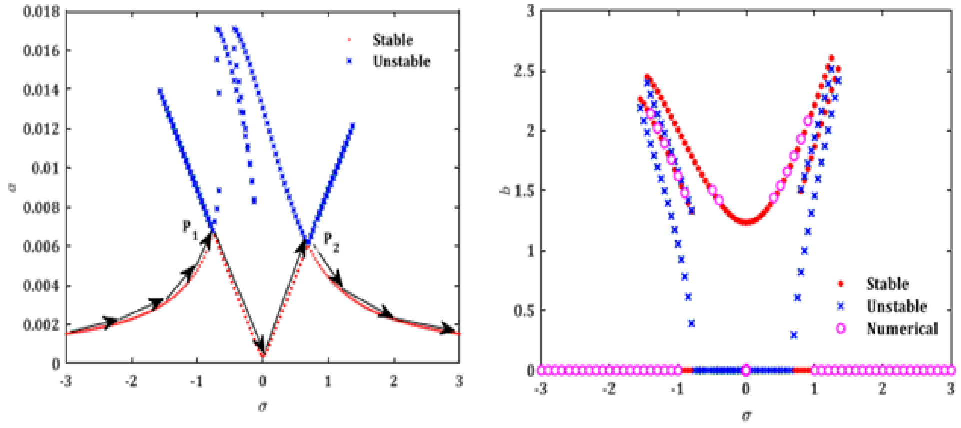
Frequency response curves for asymmetrical AMB-rotor system with PPF

16.2.Nonlinear vibration control using NSC



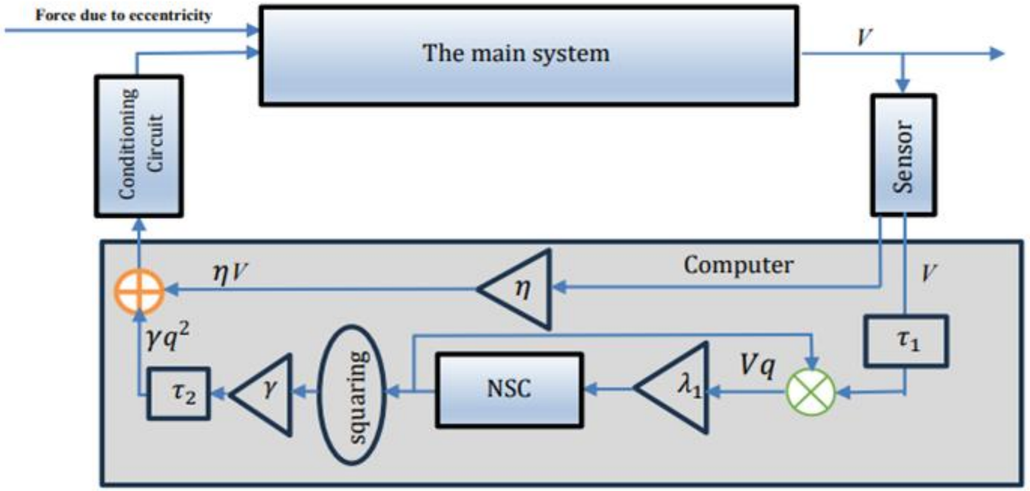
(a) (b)

Frequency response curves for symmetrical AMB-rotor system with NSC



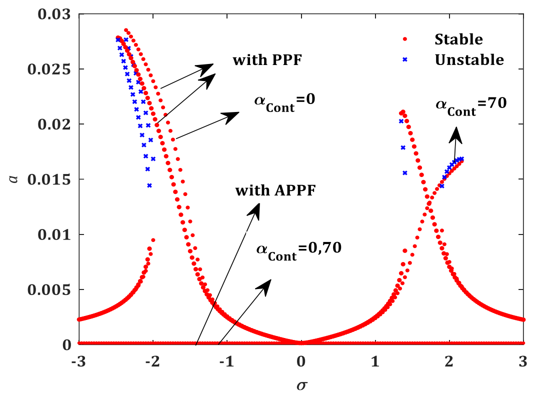
(a) (b)

Frequency response curves for asymmetrical AMB-rotor system with NSC



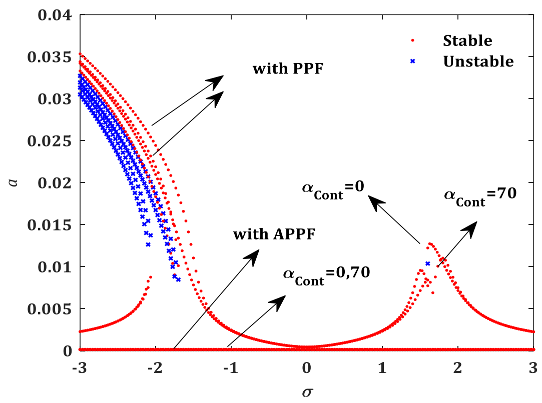
An NSC mechanism

16.3.Nonlinear vibration control using APPF



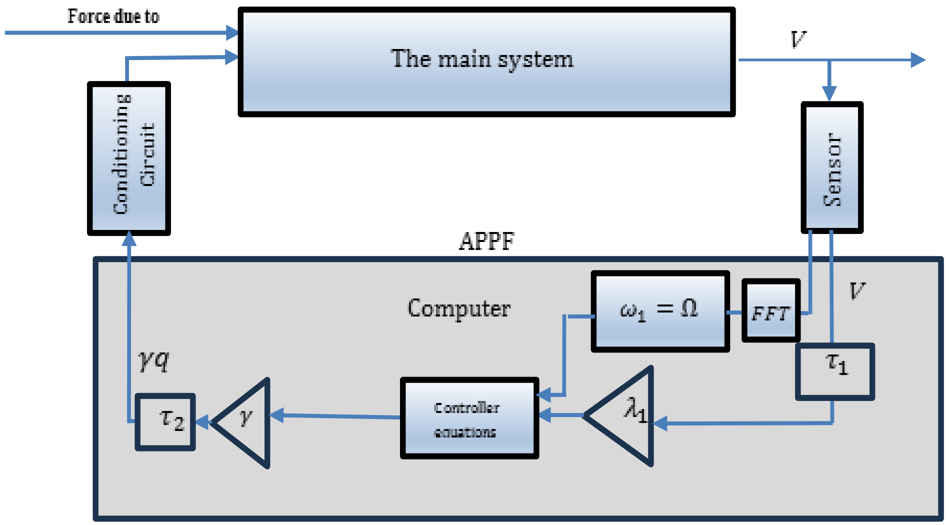
(a)

Frequency response curves for symmetrical AMB-rotor system with PPF



(b)

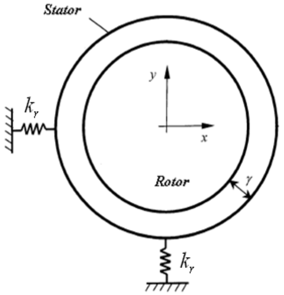
Frequency response curves for asymmetrical AMB-rotor system with PPF

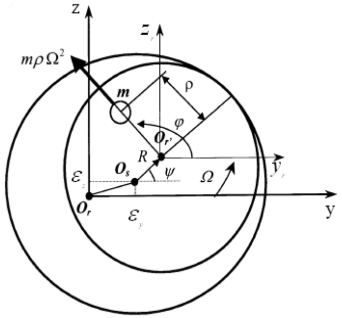


An APPF control mechanism

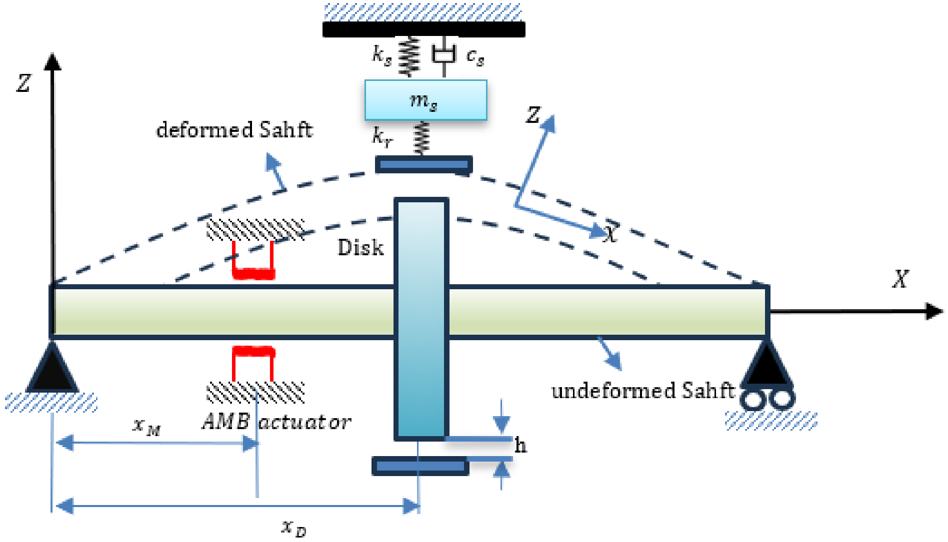
17.Chaos and control of chaos in AMB-rotor system

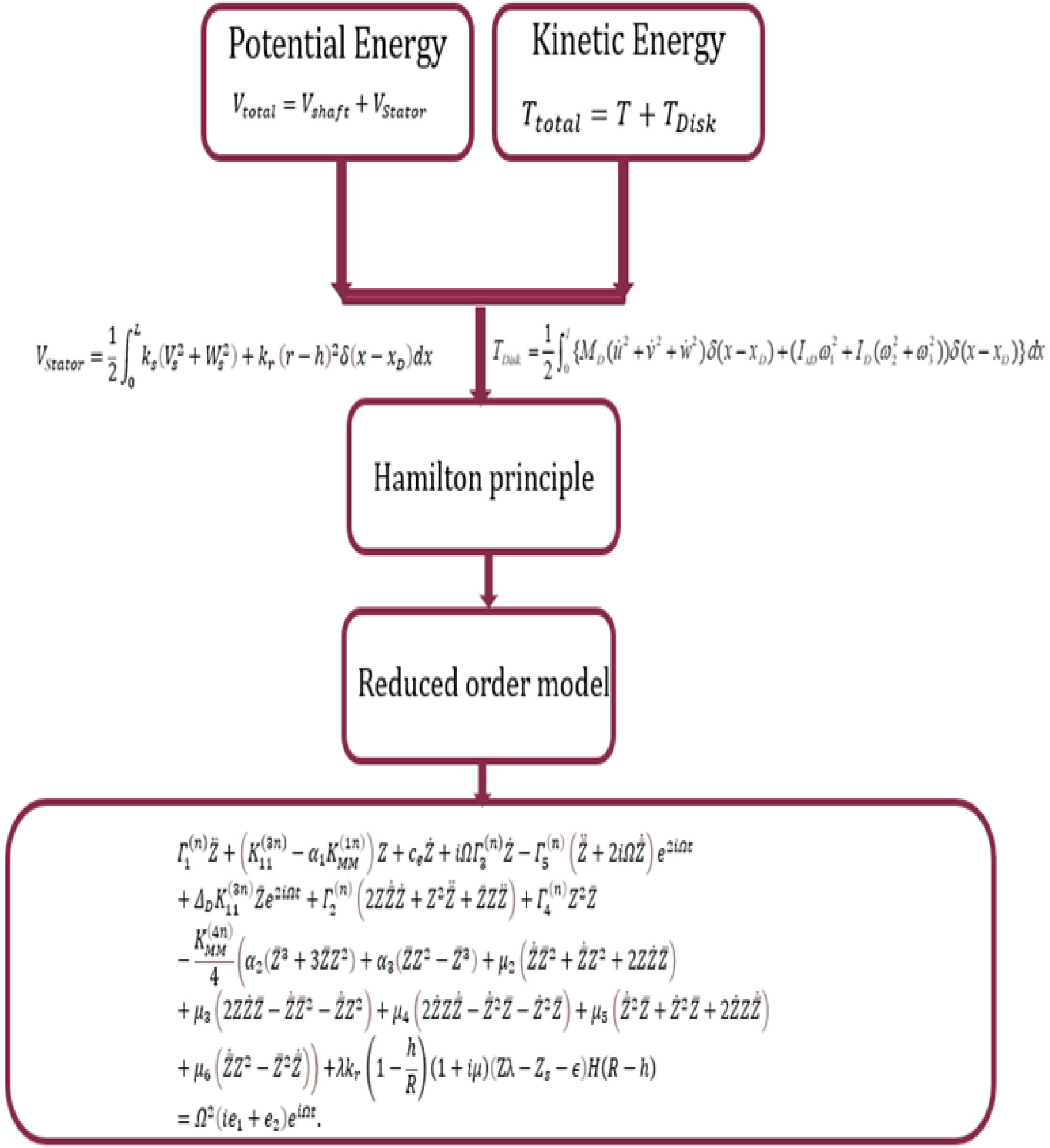
Baptiste Joseph Fourier

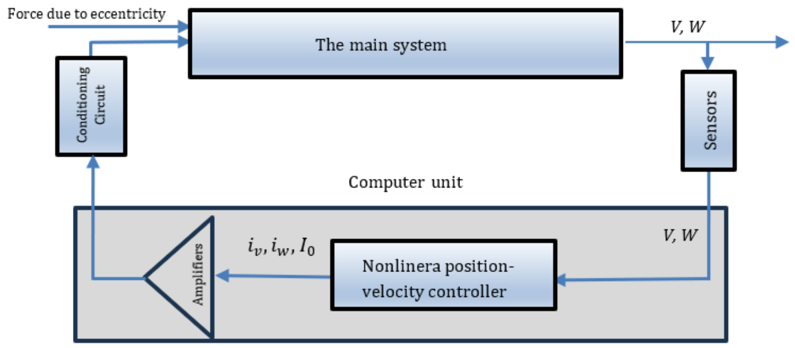




Henri Poincaré

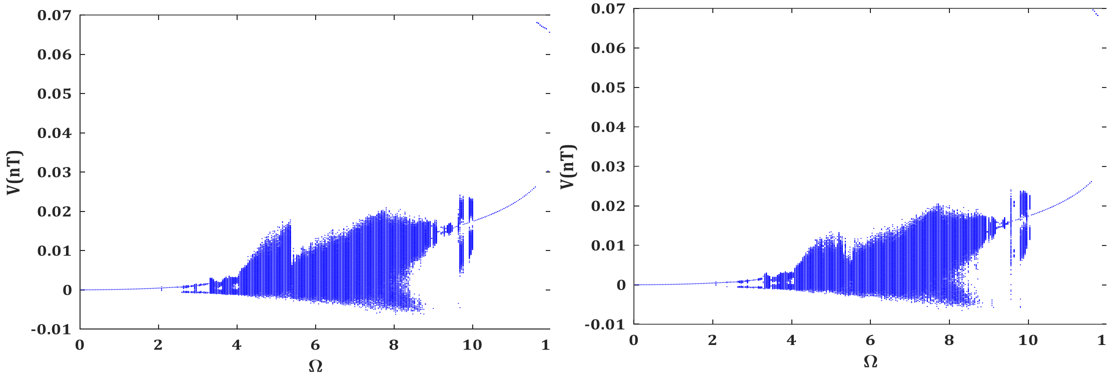




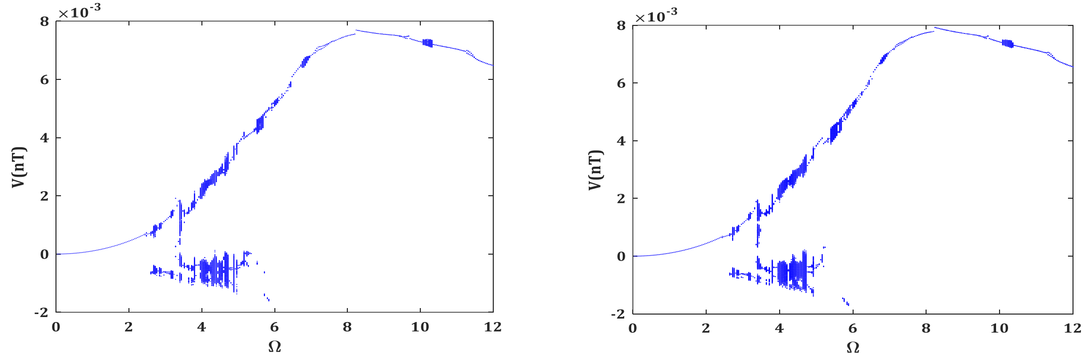


$$
\begin{aligned}
&\Gamma_1^{(n)}\ddot{Z} + \left(K_{11}^{(3n)} - \alpha_1 K_{MM}^{(1n)}\right)Z + c_e\dot{Z} + i\Omega\Gamma_3^{(n)}\dot{Z} - \Gamma_5^{(n)}\left(\ddot{\bar{Z}} + 2i\Omega\dot{\bar{Z}}\right)e^{2i\Omega t} + \Delta_D K_{11}^{(3n)}\bar{Z}e^{2i\Omega t} \\
&+ \Gamma_2^{(n)}\left(2Z\dot{Z}\dot{\bar{Z}} + Z^2\ddot{\bar{Z}} + \bar{Z}Z\ddot{Z}\right) + \Gamma_4^{(n)}Z^2\bar{Z} \\
&- \frac{K_{MM}^{(4n)}}{4}\Big(\alpha_2(\bar{Z}^3 + 3\bar{Z}Z^2) + \alpha_3(\bar{Z}Z^2 - \bar{Z}^3) + \mu_2\left(\dot{\bar{Z}}\bar{Z}^2 + \dot{Z}Z^2 + 2Z\dot{Z}\bar{Z}\right) \\
&+ \mu_3\left(2Z\dot{Z}\bar{Z} - \dot{\bar{Z}}Z^2 - \dot{Z}Z^2\right) + \mu_4\left(2\dot{Z}Z\dot{\bar{Z}} - \dot{\bar{Z}}^2\bar{Z} - \dot{Z}^2\bar{Z}\right) \\
&+ \mu_5\left(\dot{Z}^2\bar{Z} + \dot{Z}^2\bar{Z} + 2\dot{Z}Z\dot{\bar{Z}}\right) + \mu_6\left(\dot{Z}Z^2 - \bar{Z}^2\dot{Z}\right) + \mu_7\left(\dot{Z}^3 + 3\dot{Z}\dot{\bar{Z}}^2\right)\Big) + \lambda k_r(1 \\
&- \frac{h}{R})(1 + i\mu)(Z\lambda - Z_s - \epsilon)H(R - h) = \Omega^2(ie_1 + e_2)e^{i\Omega t}.
\end{aligned}
$$

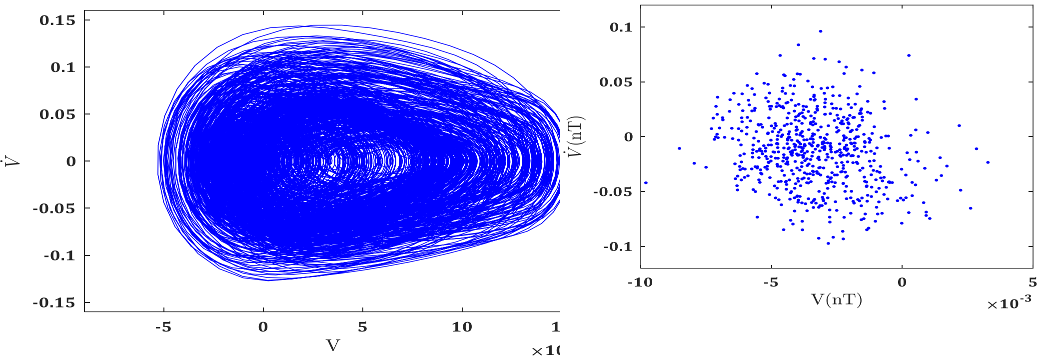
$$
\ddot{Z}_s + k_s Z_s + c_s\dot{Z}_s = \beta\lambda k_r(1 - \frac{h}{R})(1 + i\mu)(Z\lambda - Z_s - \epsilon)H(R - h)
$$



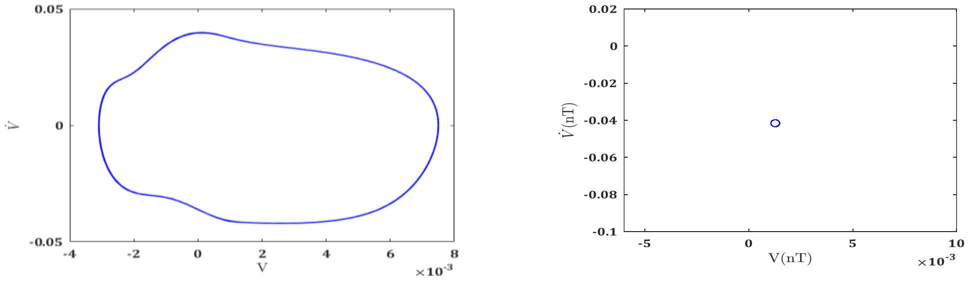
(a) (b)



(c) (d)



(a) (b)



(c) *(d)*

the effect of a nonlinear velocity-position controller on the behavior of a rotor, undergoing rotor-stator contact

Conclusion

Rotor dynamics is a cornerstone of modern machinery design and maintenance, providing essential insights into the behavior of rotating systems.

1. **Enhancing Reliability:**
    1. Mitigating critical speeds, vibrations, and resonances for safe operation.
2. **Improving Efficiency:**
    1. Leveraging advanced modeling techniques for optimized machinery performance.
3. **Driving Innovation:**
    1. Utilizing technologies like Active Magnetic Bearings (AMB) and control mechanisms to handle both regular and irregular rotor behaviors.

Control Mechanisms for Rotor Behavior:

- **NSC (Nonlinear Saturation Controller):** mitigating large amplitude of vibrations.
- **Parametric Controllers:** Dynamically adapts system parameters for irregular behavior.
- **PPF (Positive Position Feedback):** Controls resonance and improves stability.
- **APPF (Adaptive PPF):** Expands PPF for a wider range of speeds.
- **NPVC (Nonlinear Position-Velocity Controller):** Controls chaos effectively.

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[1] M. Shahgholi and S. E. Khadem, “Primary and parametric resonances of asymmetrical rotating shafts with stretching nonlinearity,” *Mech Mach Theory*, vol. 51, pp. 131–144, May 2012, doi: 10.1016/J.MECHMACHTHEORY.2011.12.012.

[2] M. Shahgholi and S. E. Khadem, “Internal, combinational and sub-harmonic resonances of a nonlinear asymmetrical rotating shaft,” *Nonlinear Dyn*, vol. 79, no. 1, pp. 173–184, Jan. 2015, doi: 10.1007/S11071-014-1654-0/FIGURES/11.

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Thank you

Thank you for your time and attention! I hope you found this presentation insightful