Properties of the electromagnetic softening and hardening spring: experiment and simulation

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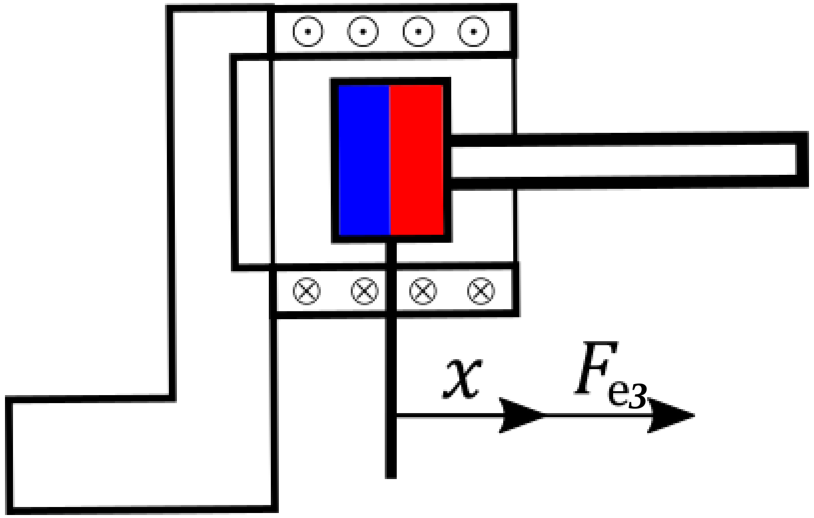
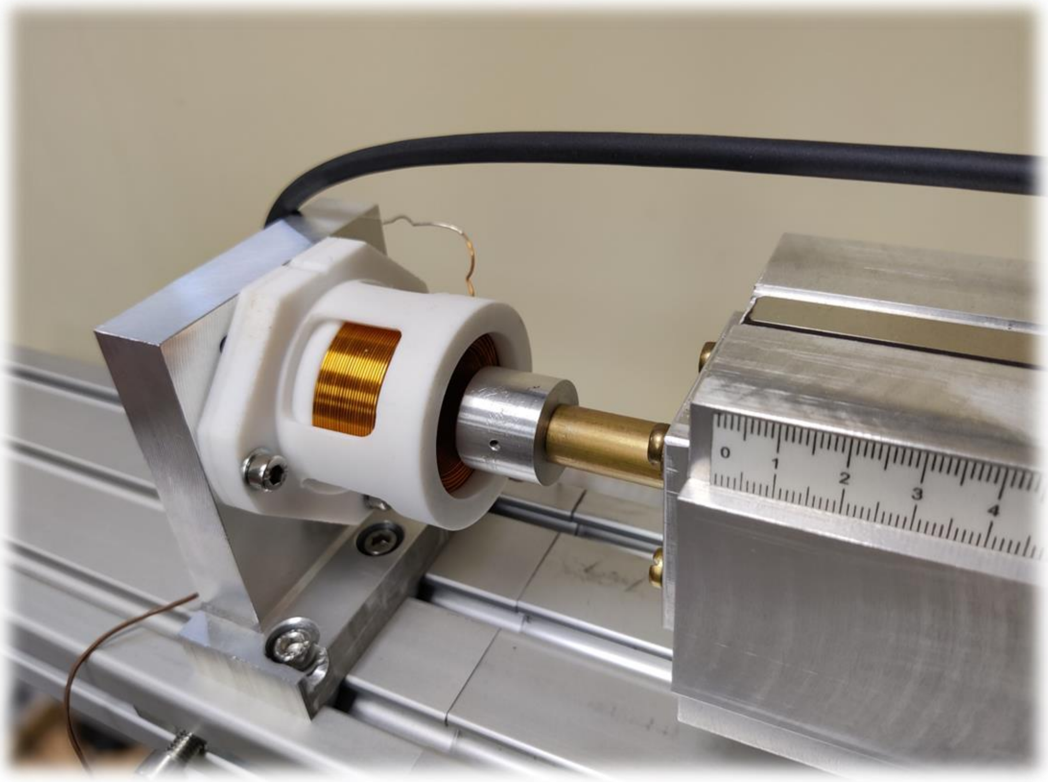
14.05.2024

Electromagnetic spring

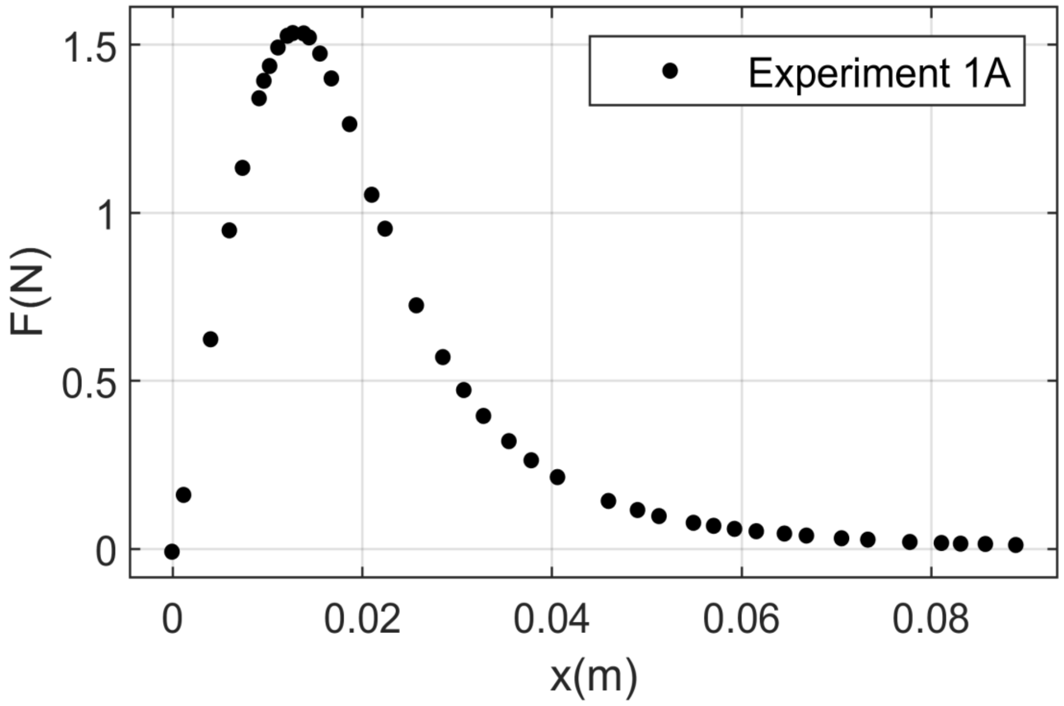
What is an electromagnetic spring ?





Electromagnetic spring

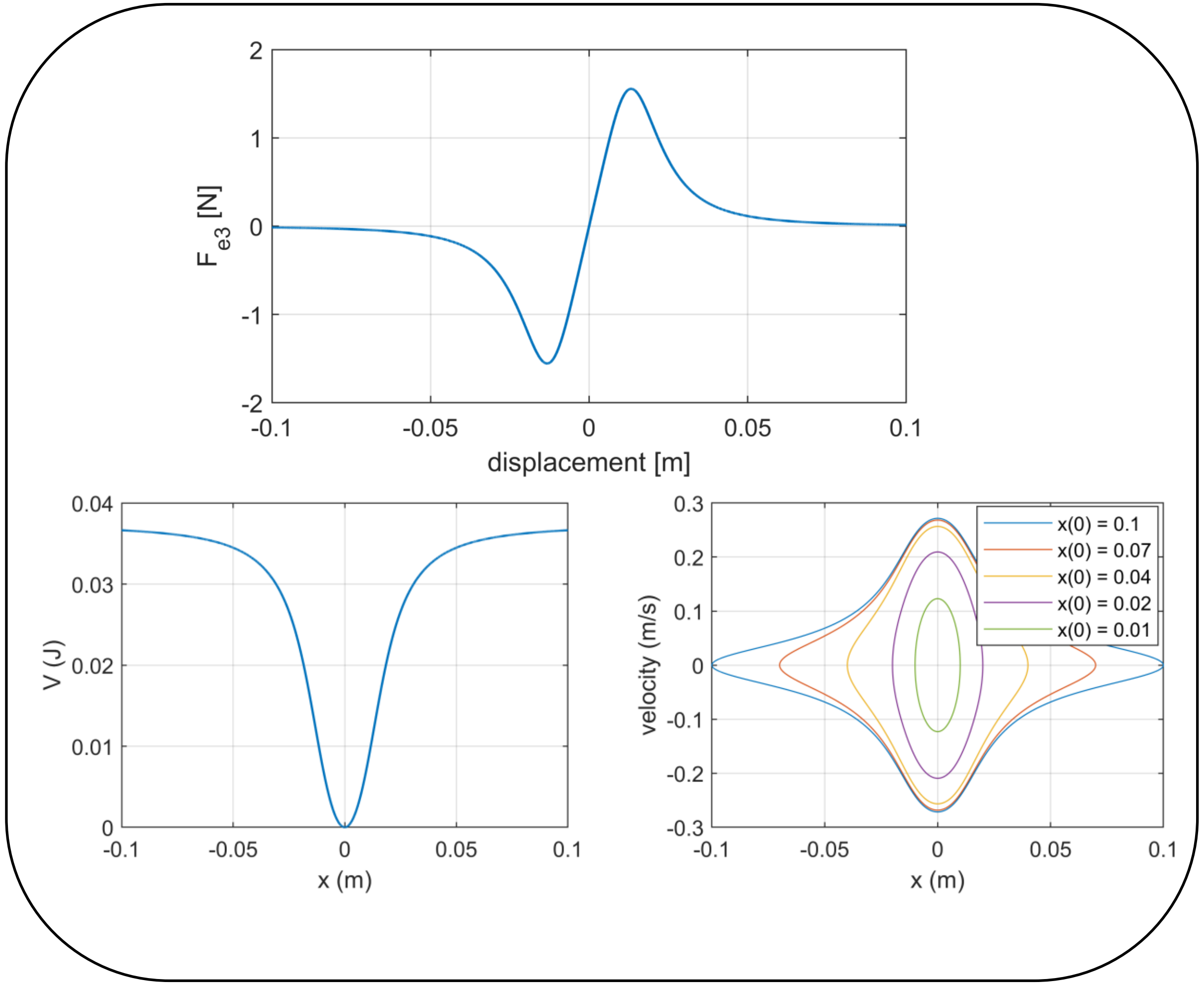
How to mathematically describe the force?

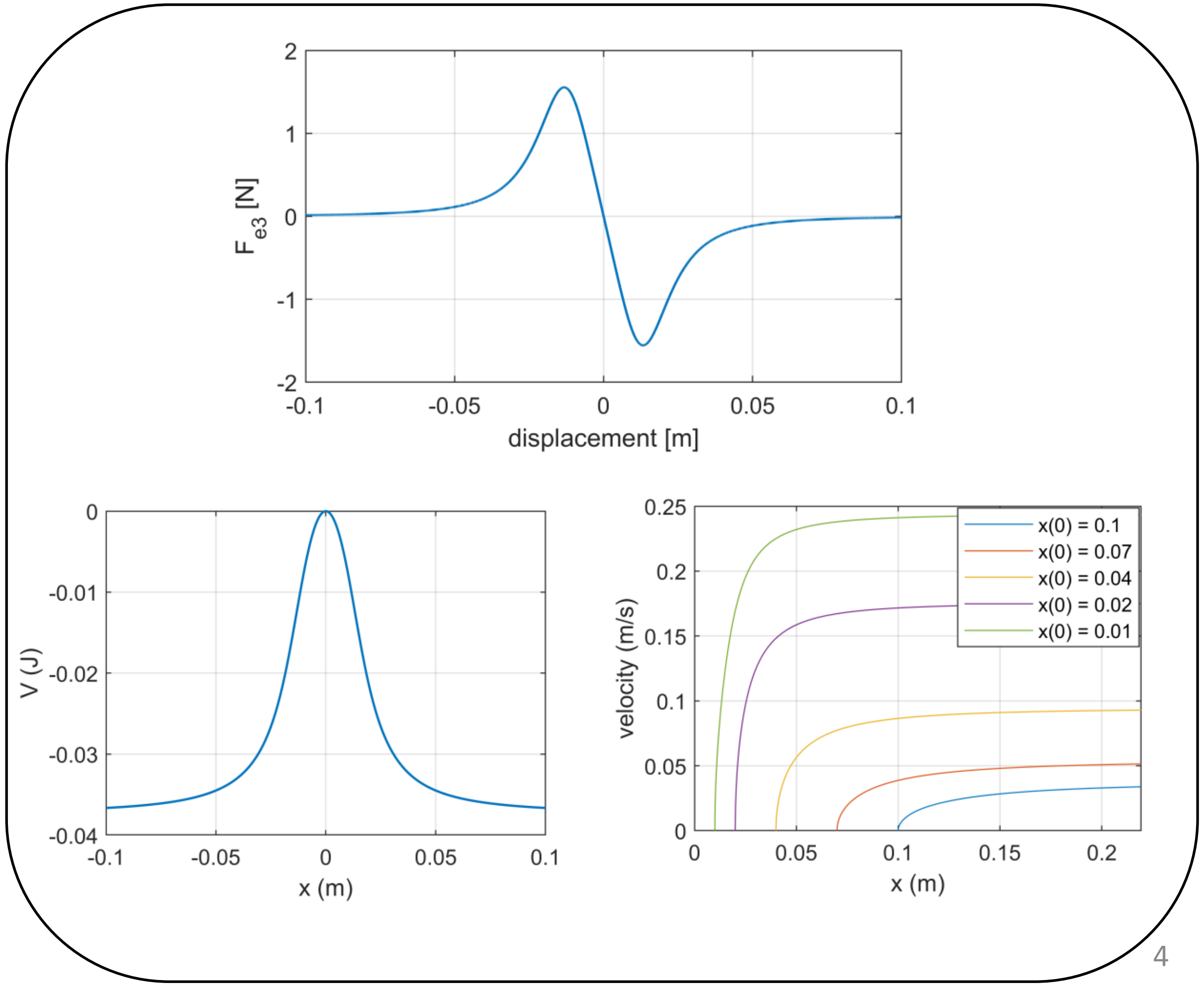


$$F_{e3} = \frac{pIq^3x}{q^4 + x^4}$$

Electromagnetic spring

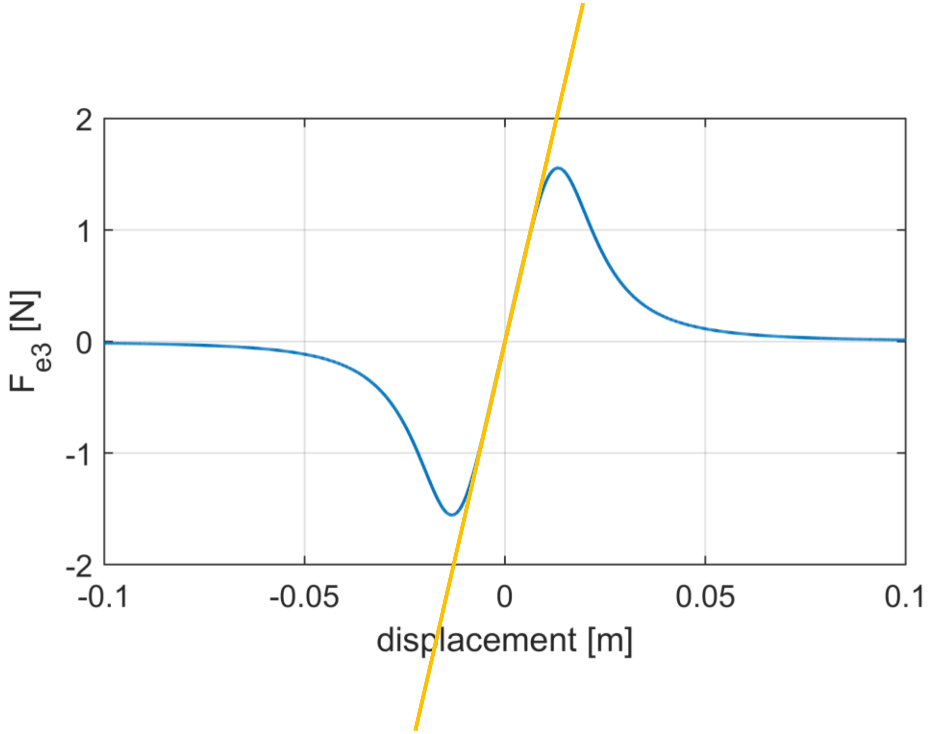
Polarity of the flowing current

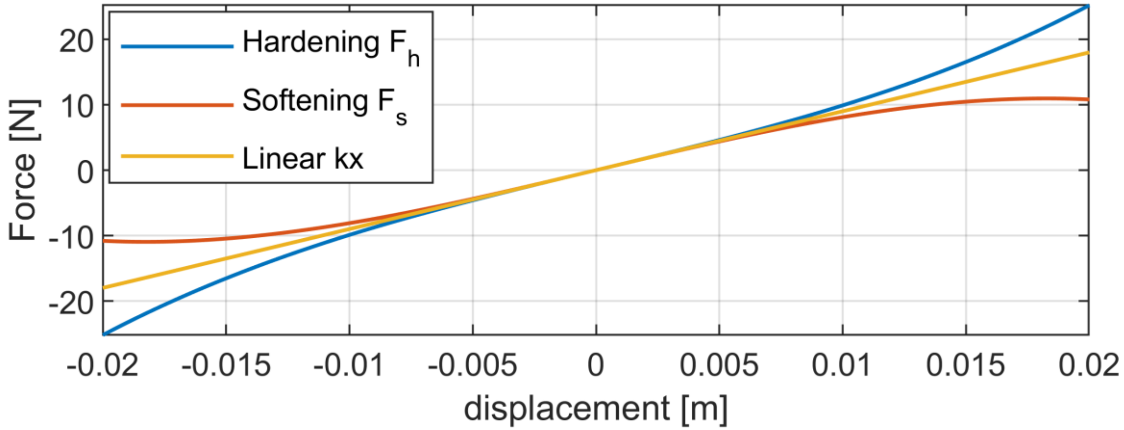




Nonlinearity in springs

Softening and hardening characteristic



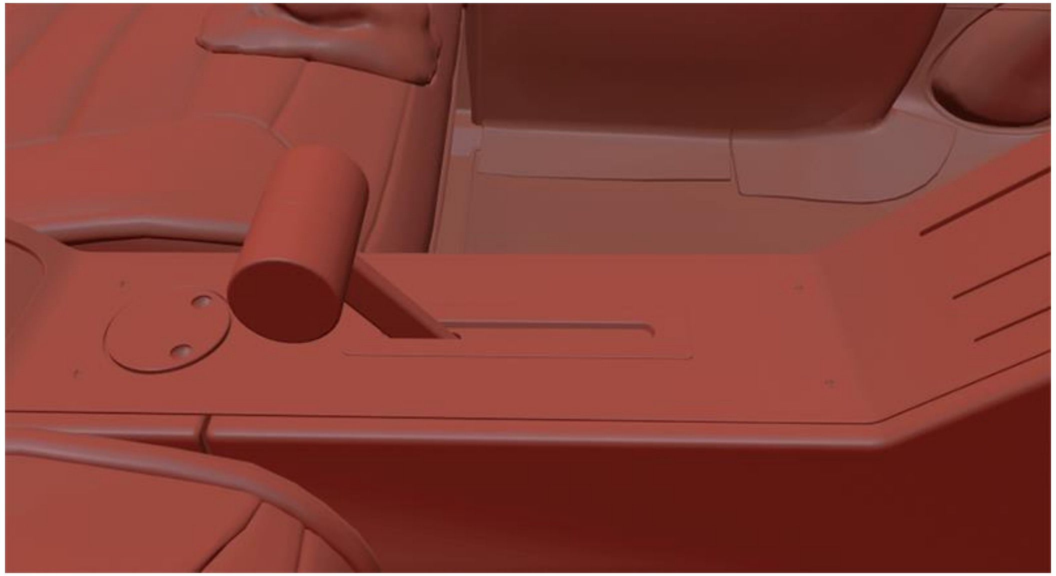


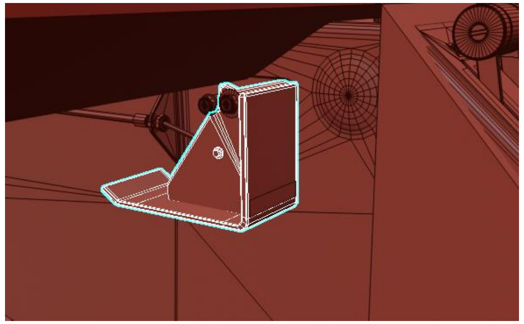
Hardening springs are such kind of nonlinear springs in which the force increases more rapidly than the deformation. Softening springs are such a kind of a springs in which the resistive force gets smaller as the spring deflection becomes larger [1].

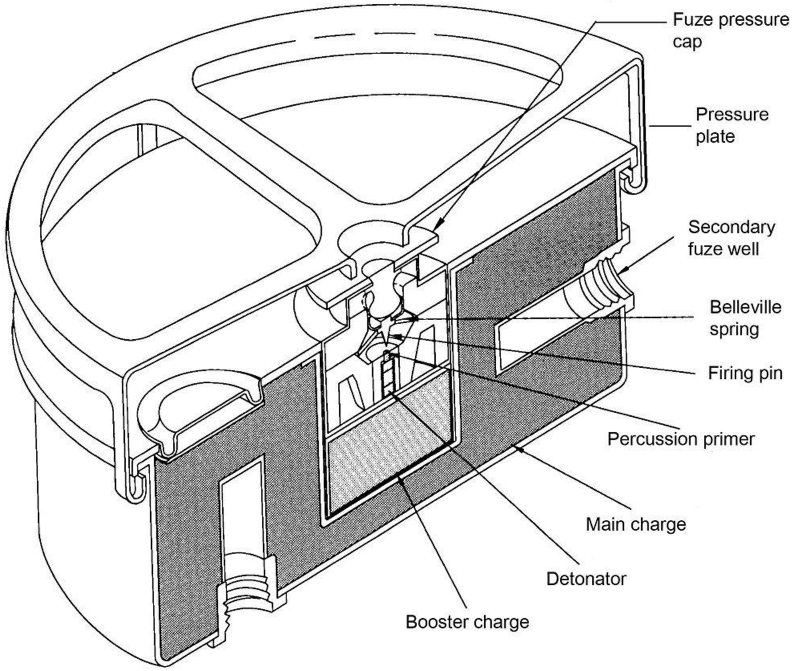
[1] Haim Baruh, Applied Dynamics, CRC Press, 2014

Nonlinearity in springs

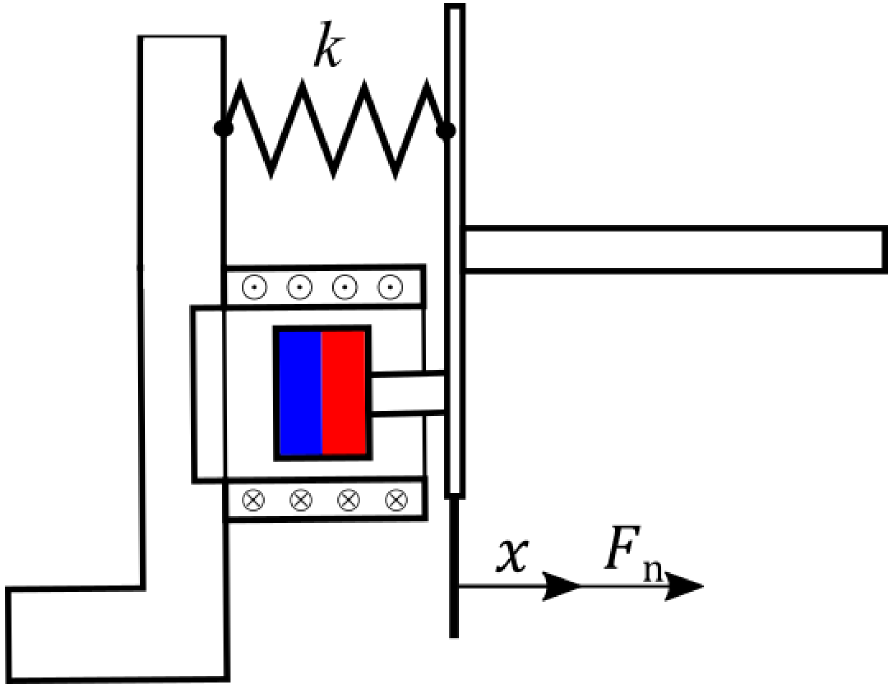
Applications of softening and hardening characteristic







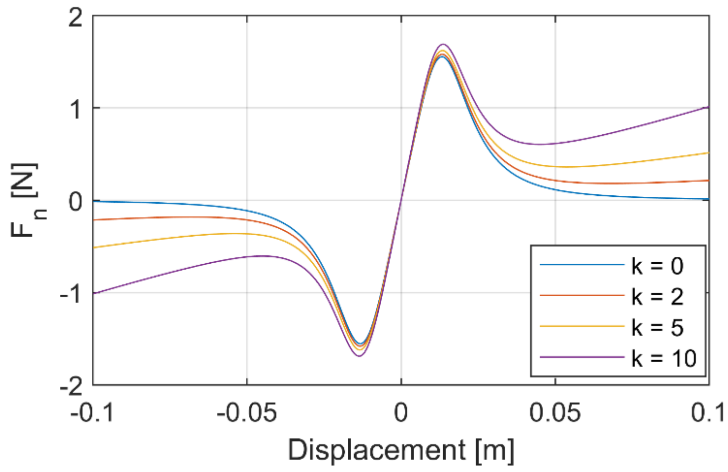
Combined stiffness

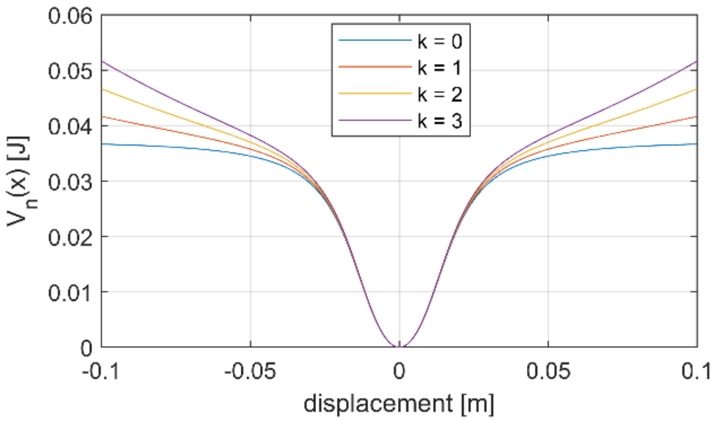


$$F_n(x) = kx + \frac{Ipq^3}{q^4 + x^4} \cdot x$$

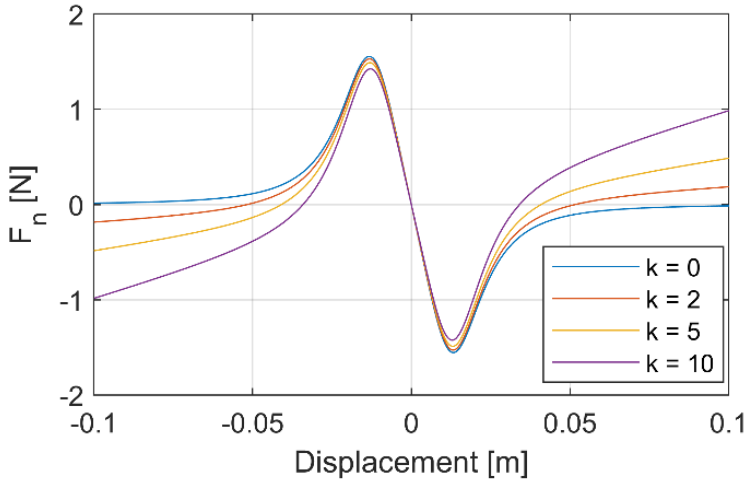
Combined stiffness

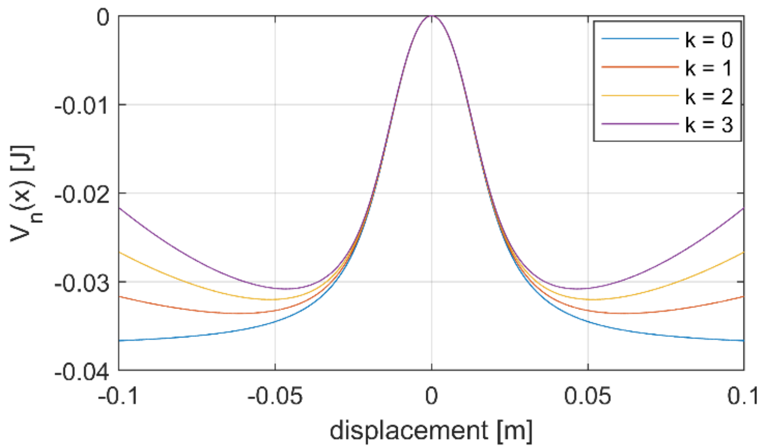
I = 1 A





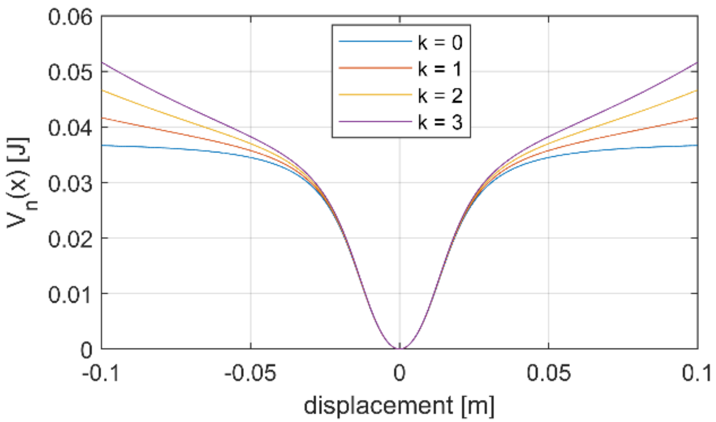
I = -1 A

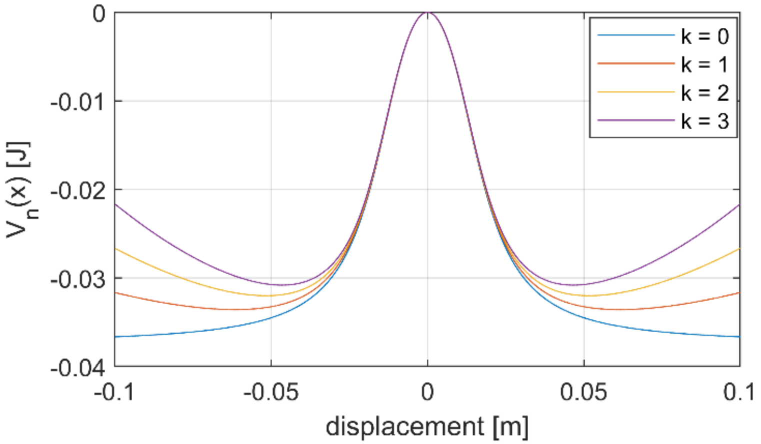




Combined stiffness

Equilibrium position



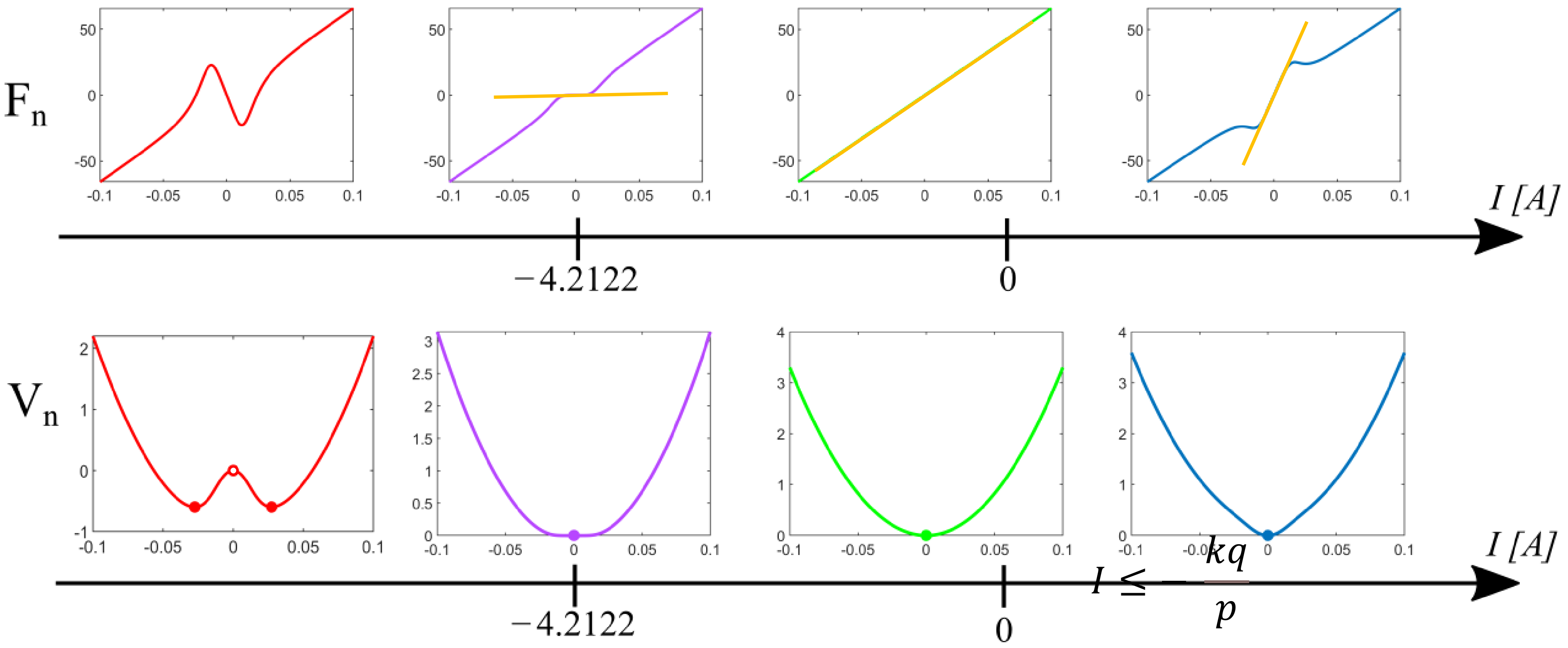


$$\begin{cases} \dot{x}_1 = 0 \\ -\dfrac{\mathrm{k}}{\mathrm{m}_1} x_1 - \dfrac{Ipq^3}{m_1\left(q^4 + {x_1}^4\right)} \cdot x_1 = 0 \end{cases}$$

$$x_1 = 0 \qquad \vee \qquad x_1 = \pm \sqrt[4]{\frac{-kq^4 - Ipq^3}{k}}$$

$$I \leq -\frac{kq}{p}$$

Combined stiffness



$$I \leq -\frac{kq}{p}$$

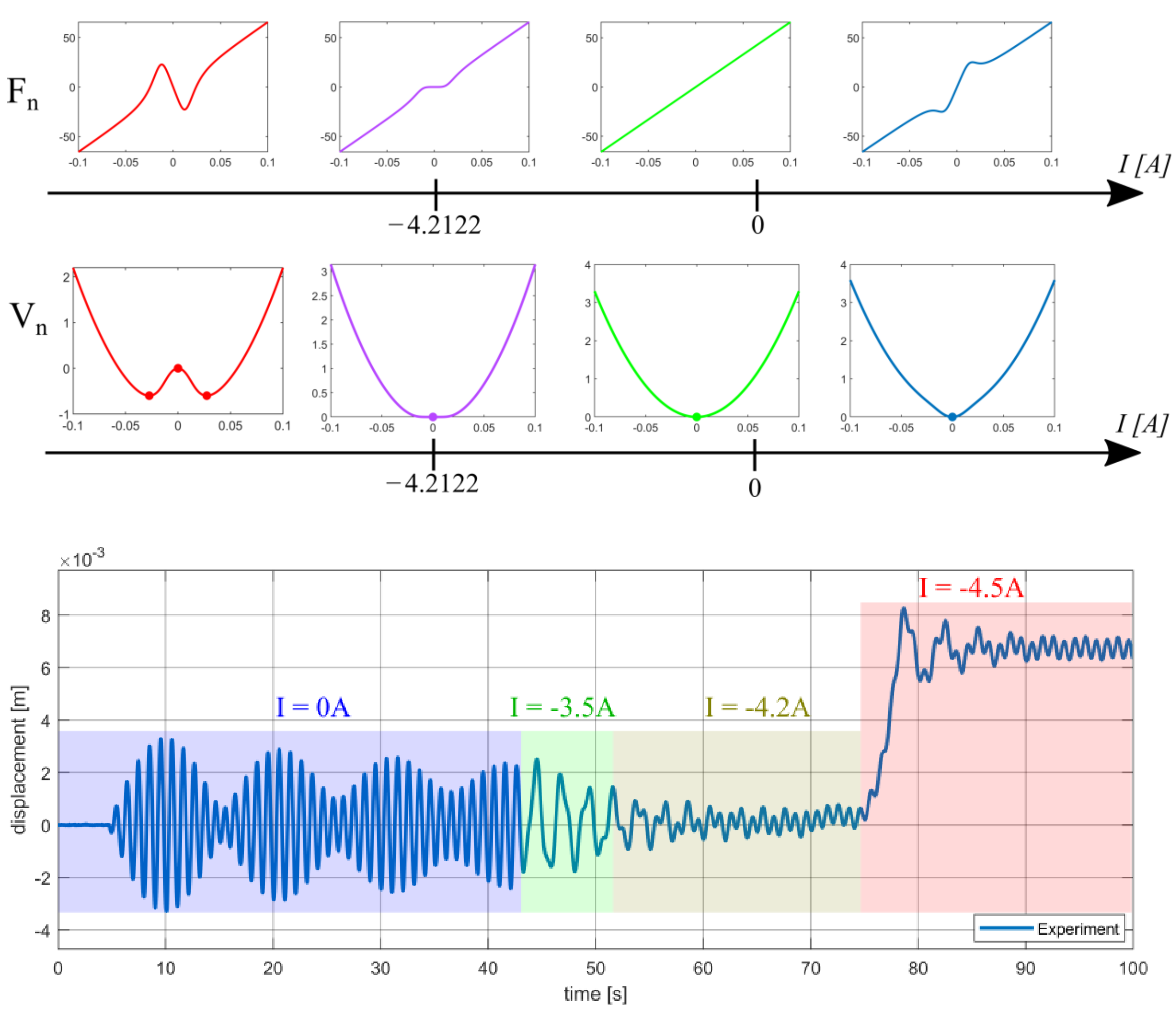
$k = 659.5\,\frac{N}{m}$

$p = 2.729\,\frac{N}{A}$

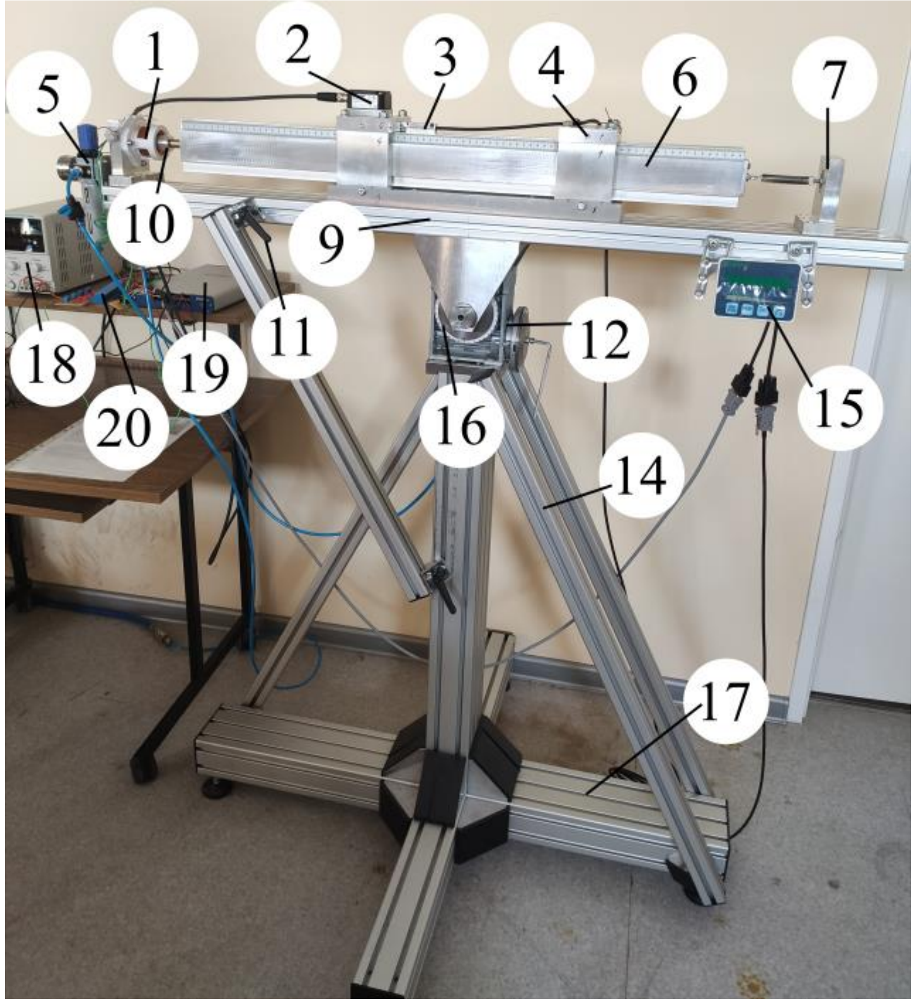
$q = 0.01743\ m$

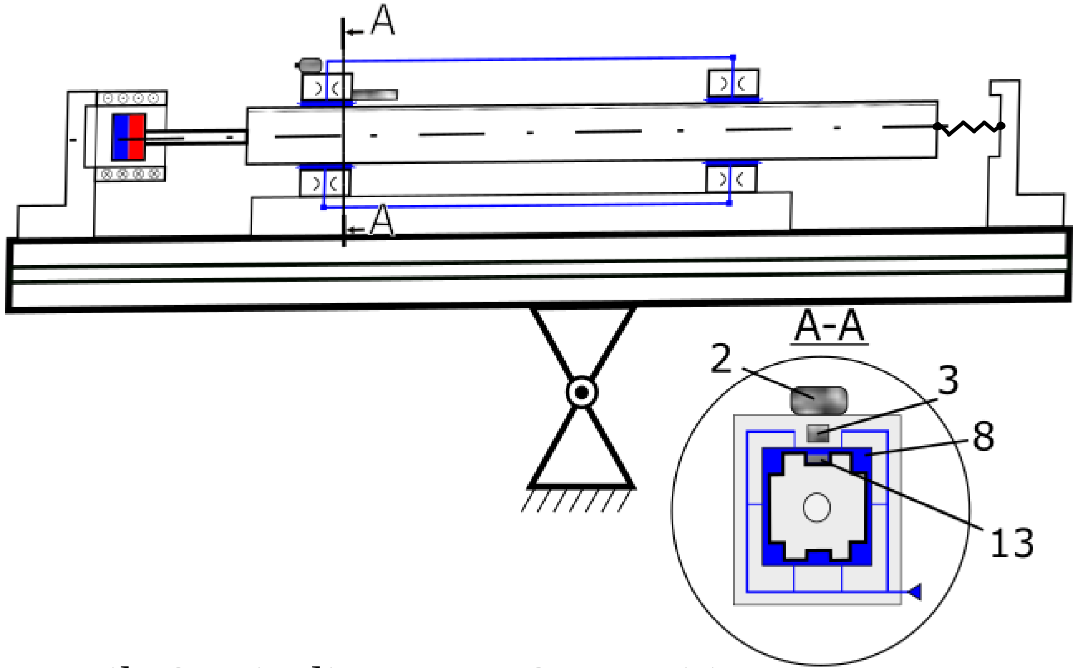
$$I = -4.2122\ [A]$$

Combined stiffness



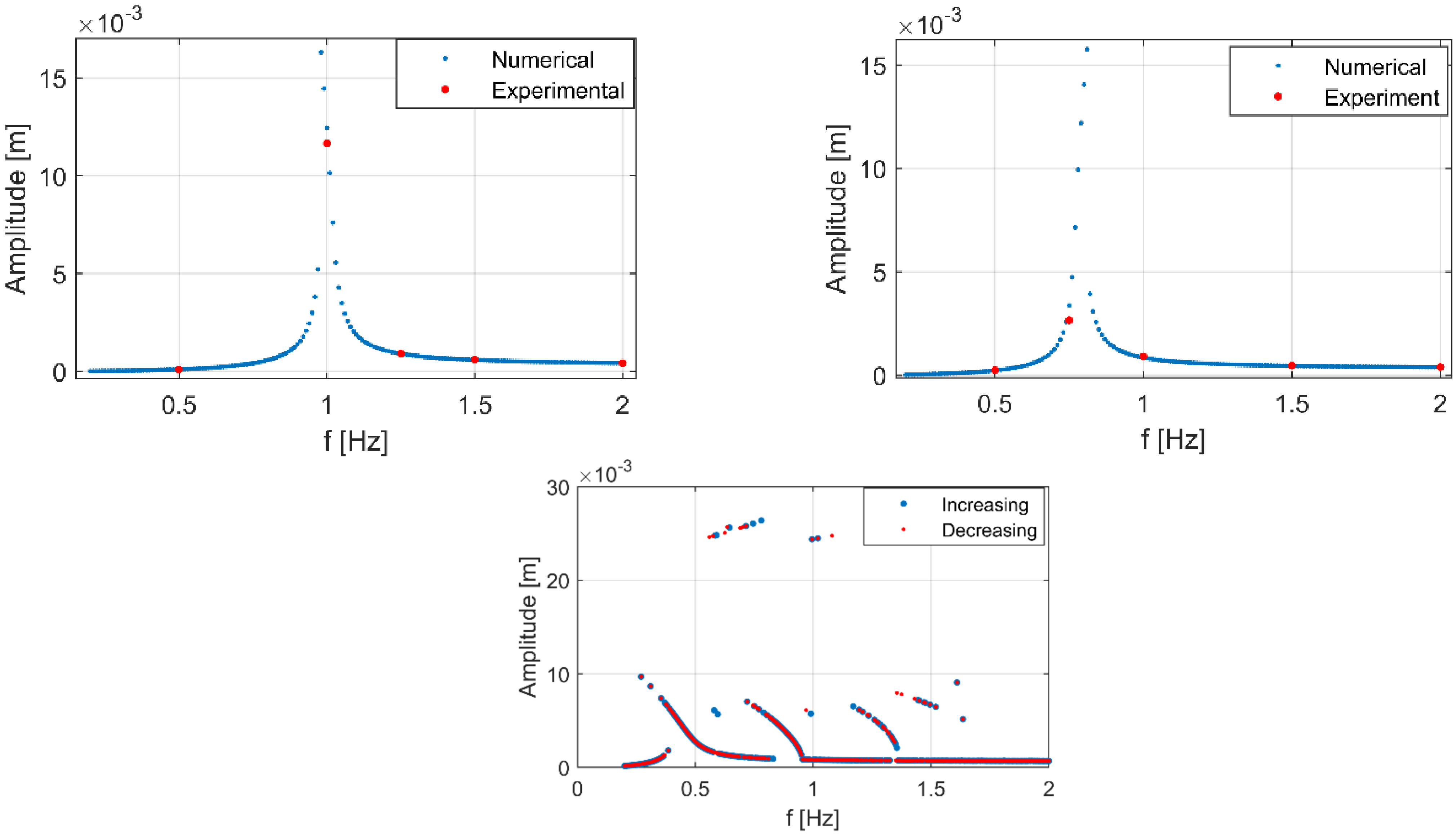
Experimental rig





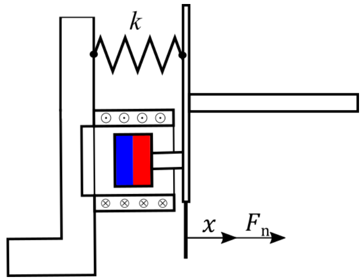
1 - coil, 2 - inclinometer, 3 - position sensor, 4 - aerostatic supports, 5 - air preparation system, 6 - slider, 7 - brackets, 8- compressed air, 9 - base , 10 - magnet, 11 - angle adjustment locks , 12 - worm gear, 13 - magnetic ruler, 14 - supports, 15 - displacement display, 16 –angular scale, 17 - rig frame, 18 - power supply, 19 - data acquisition card, 20 – H-bridge.

Frequency responce

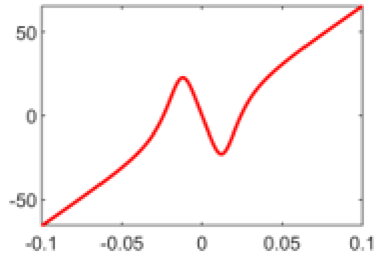


Concluding remarks

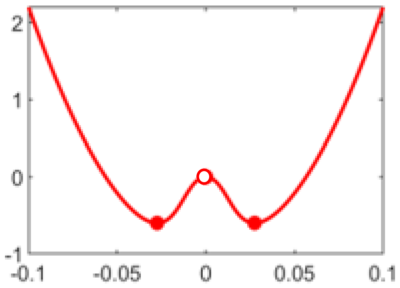
- Electromagnetic spring force has been modeled and validated experimentally



- Spring of combined stiffness can produce softening, hardening, linear and bistable stiffness

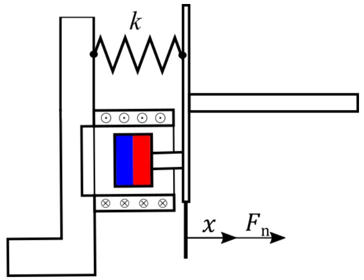


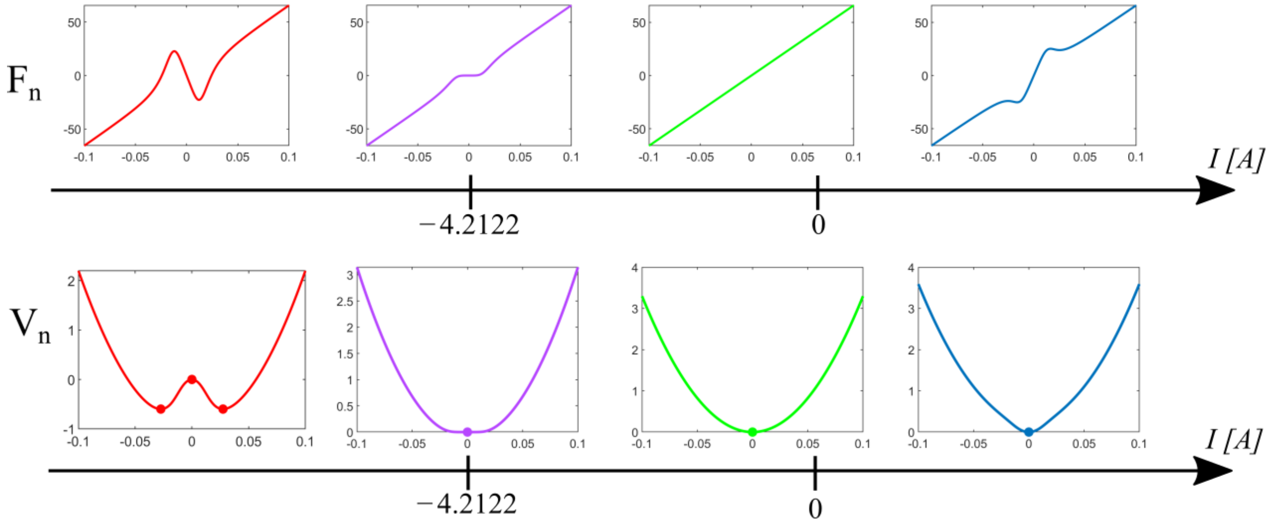
- The spring can reproduce stiffness similar to Duffing stiffness $F_d = \alpha x + \beta x^3$

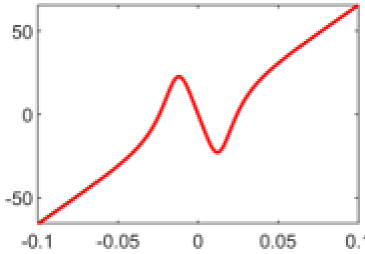


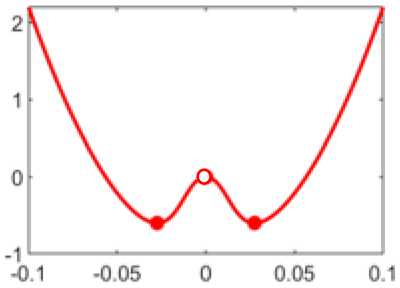
Thank you for your attention

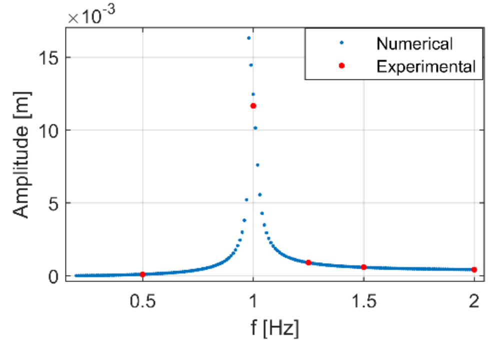
- Spring of combined stiffness can produce softening, hardening, linear and bistable stiffness
- The spring can reproduce stiffness similar to Duffing stiffness

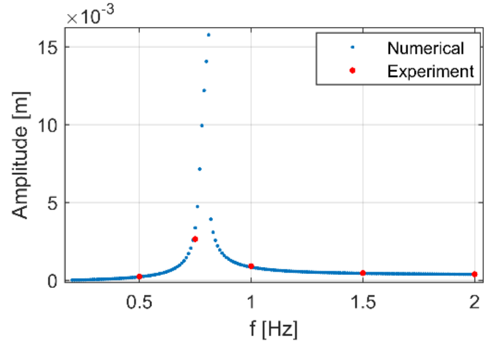


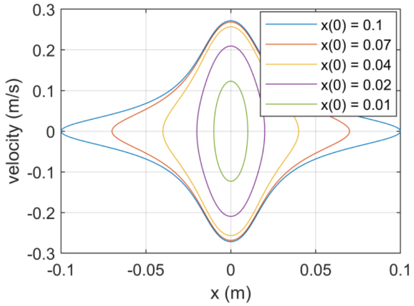


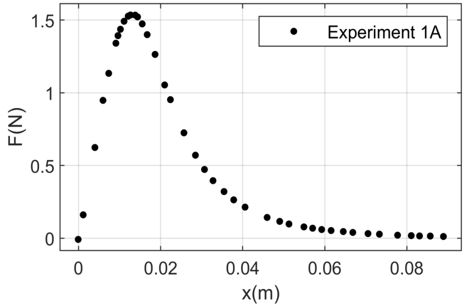












Extention of the presentation

$$m_1\ddot{x}_1 = -F_k(x_1) - F_e(x_1) - F_{c3}(\dot{x}_1) + F_h(t)$$

$$m_1\ddot{x}_1 = -kx_1 - \frac{Ipq^3}{q^4 + {x_1}^4} \cdot x_1 - c_{c1}\dot{x}_1 + c_{c2}{\dot{x}_1}^3 + c_{c3}\mathrm{sgn}(\dot{x}_1) + 2m_u r\omega^2 \sin(\omega t)$$