Lodz University of Technology
Department of Automation, Biomechanics and Mechatronics

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Dynamics of pendulum forced by a magnetic excitation with position-dependent phase

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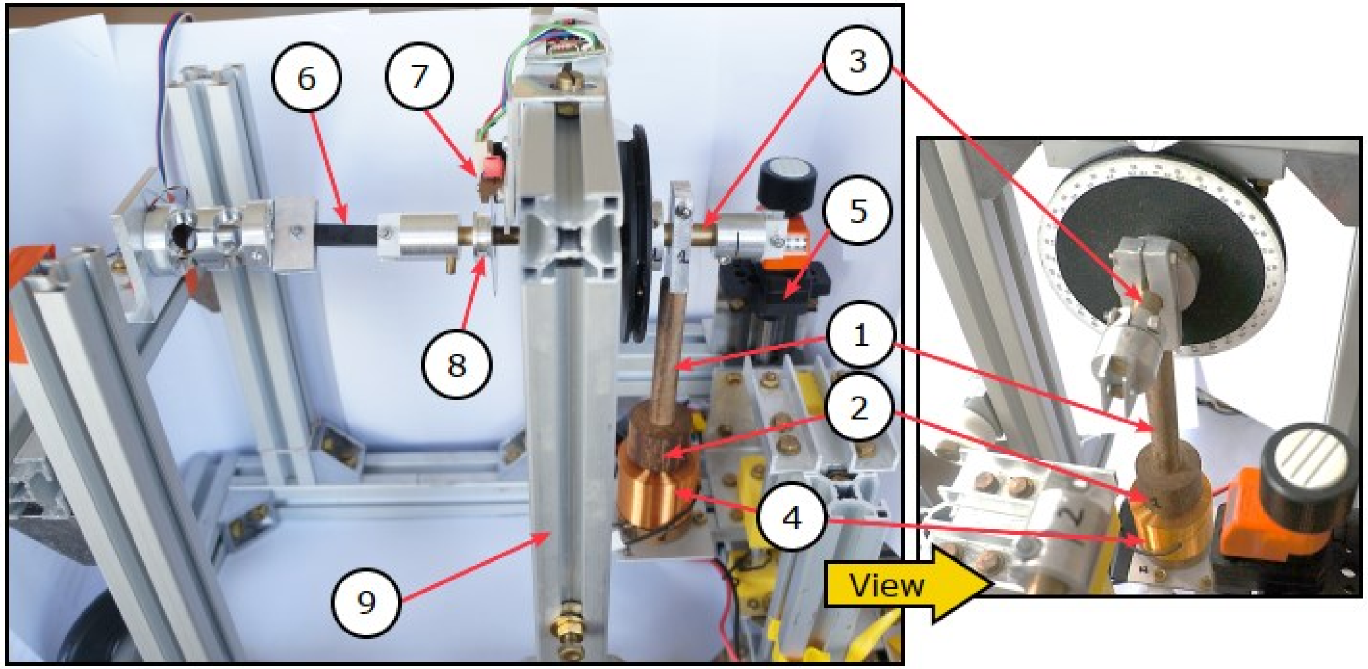
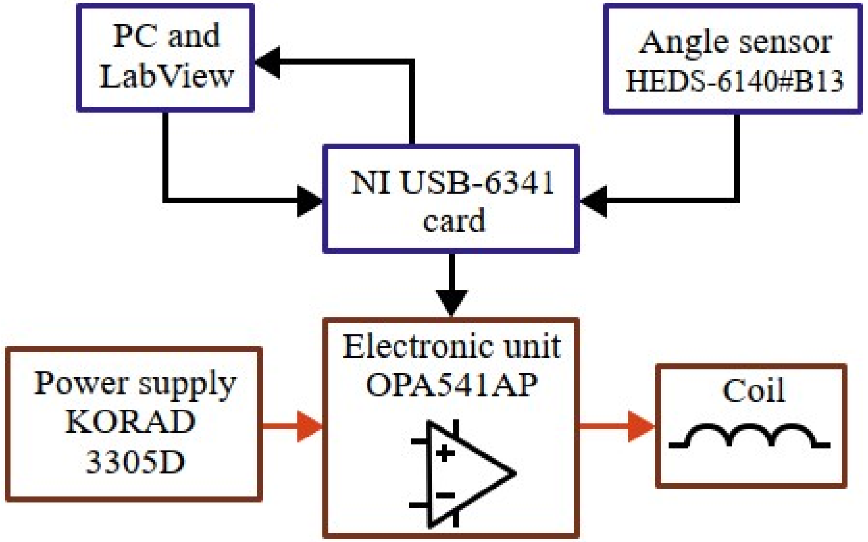
Agenda

1. Experimental system
2. Mathematical modeling
3. Validation of the model
4. System under excitation with a controlled phase
5. Concluding remarks

Experimental system



(a)



(b)

Figure 1: Experimental setup (a): (1) – pendulum, (2) – neodymium magnet, (3) – brass axis, (4) – electric coil, (5) – linear lift, (6) – an elastic element, (7) – optical sensor, (8) – code wheel, (9) – aluminium frame.
Signal-flow diagram (b), red arrows indicate current signals whereas black arrows indicate voltage signals.

Mathematical modeling

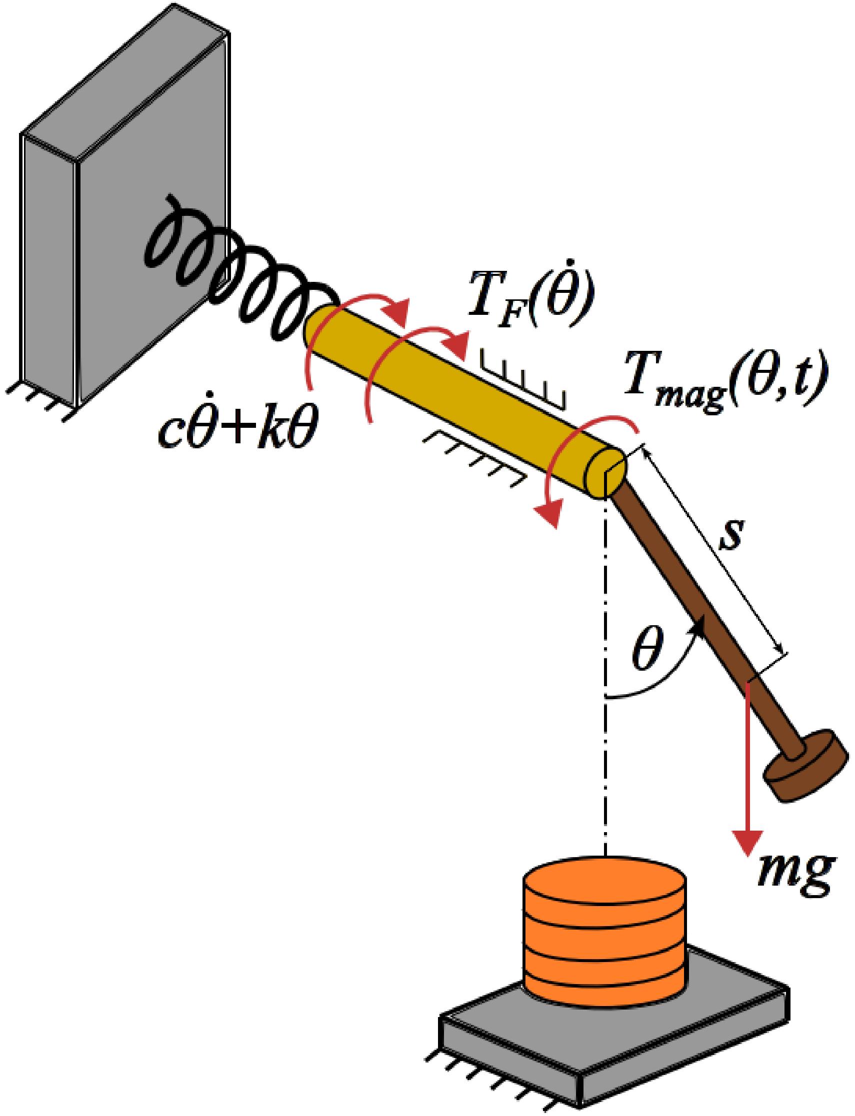


Figure 2: Scheme of the studied system.

$$J\ddot{\theta} + c\,\dot{\theta} + mgs\sin\theta + k\,\theta + T_F(\dot{\theta}) = T_{mag}(\theta, i(t)) \qquad (1)$$

where:
θ – pendulum angular position,
J – moment of inertia
k – a stiffness of the elastic joint
c – total viscous damping coefficient.
mg – a gravitational force
s – distance of center of mass from the axis of rotation.

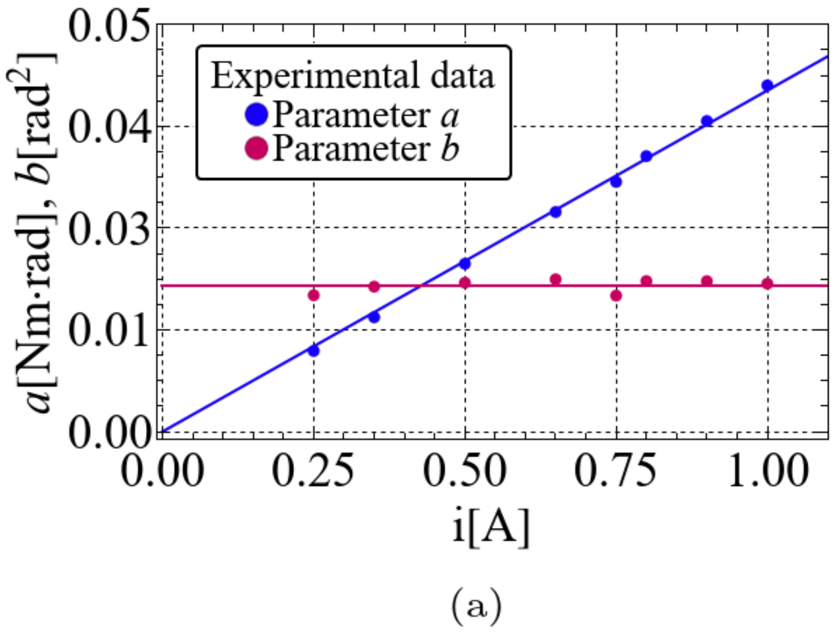
$$T_F(\dot{\theta}) = \left[\tau_c + (\tau_s - \tau_c)\exp\left(\frac{-\dot{\theta}^2}{v_s^2}\right)\right]\tanh \epsilon\dot{\theta} \qquad (2)$$

where:
τ_c, τ_s – Coulomb friction and static friction torques
v_s – Stribeck velocity coefficient
ϵ – regularization parameter

Mathematical modeling

$$T_{mag}(\theta, i(t)) = \frac{2a(i(t))}{b(i(t))} \exp\left[\frac{-\theta^2}{b(i(t))}\right]\theta \qquad (3)$$

where: $a(i(t))$, $b(i(t))$ – parameters responsible for magnitude and shape of the magnetic torque



$$a(i(t)) = K_p i(t)\Big|_{K_p = const}$$

$$b(i(t)) = const \qquad (4)$$

Figure 3: (a) Characteristics of the *a* and *b* parameters obtained for constant coil current *i(t)*.
(b) Experiment and fitted eq. (3) of the magnetic torque $T_{mag}(\theta)$ obtained for constant coil current.

Mathematical modeling

$$T_{mag}(\theta, i(t)) = \frac{2a(i(t))}{b(i(t))} \exp\left[\frac{-\theta^2}{b(i(t))}\right]\theta \qquad (3)$$

$$b(i(t)) = const$$

$$a(i(t)) = K_p i(t)\Big|_{K_p = const} \qquad (4)$$

$$i(t) = I_0 \sin(\omega_0 t + \phi_0) \qquad (5)$$

$$T_{mag}(\theta, t) = \frac{2K_p I_0}{b} \exp\left(\frac{-\theta^2}{b}\right)\theta \sin(\omega_0 t + \phi_0) \qquad (6)$$

where: I_0 – constant amplitude of the coil current
ω_0 – the angular frequency
ϕ_0 – initial phase

Mathematical modeling

$$J\ddot{\theta} + c\,\dot{\theta} + mgs\sin\theta + k\,\theta + T_F(\dot{\theta}) = T_{mag}(\theta, i(t)) \qquad (1)$$

$$T_F(\dot{\theta}) = \left[\tau_c + (\tau_s - \tau_c)\exp\left(\frac{-\dot{\theta}^2}{v_s^2}\right)\right]\tanh\epsilon\dot{\theta} \qquad (2)$$

$$T_{mag}(\theta, t) = \frac{2K_pI_0}{b}\exp\left(\frac{-\theta^2}{b}\right)\theta\sin(\omega_0 t + \phi_0) \qquad (6)$$

$$y = \frac{\theta}{\theta_s} \qquad x = \frac{t}{t_s}$$

Scaling factors:

$$\theta_s = \sqrt{b} \qquad t_s = \sqrt{\frac{J}{mgs}}$$

$$y'' + \beta y' + \alpha y + \gamma\sin\left(\frac{1}{\gamma}y\right) + \left[\delta + \zeta\exp\left(\nu y'^2\right)\right]\tanh(\sigma y') = A_0\exp(-y^2)\sin(\Omega x + \phi_0) \qquad (7)$$

where:

$$\alpha = \frac{k}{mgs}, \beta = \frac{c\sqrt{J}}{J\sqrt{mgs}}, \gamma = \frac{1}{\sqrt{b}}, \delta = \frac{F_c}{mgs\sqrt{b}}, \zeta = \frac{F_s - F_c}{mgs\sqrt{b}}, \nu = -\frac{bmgs}{v_s^2 J}, \sigma = \frac{\epsilon\sqrt{bmgs}}{\sqrt{J}}, A_0 = \frac{2K_pI_0}{bmgs}, \Omega = \frac{\omega_0}{\sqrt{mgs/J}}.$$

Validation of the model

Table 1: Values of dimensionless parameters.

Parameter	α	β	γ	δ
Value	0.34407	0.03225	7.46857	0.01956
Parameter	ζ	ν	σ	
Value	0.01786	-2.98415	5.67742	

Linear stability of the lower static equilibrium position of the pendulum

$$\begin{cases} (\theta, \dot{\theta}) = (0,0) \\ \sin(\Omega x + \phi_0) = \pm 1 \end{cases}$$

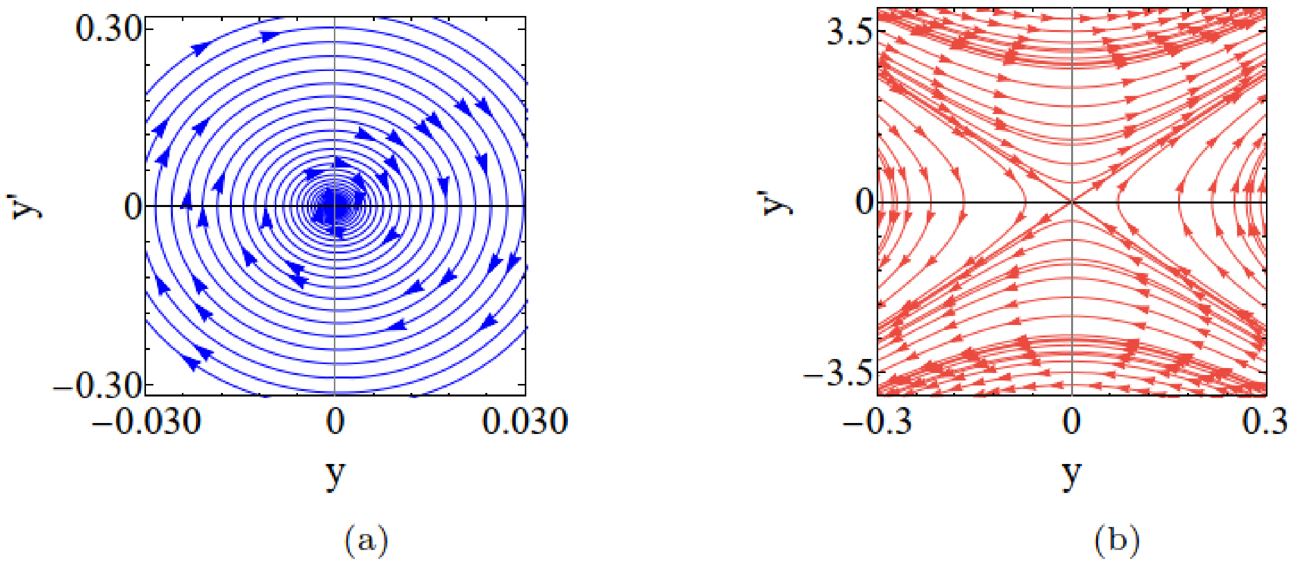


Figure 4: Stability of the lower equilibrium in terms of the coil current sign: (a) negative vs. (b) positive coil current.

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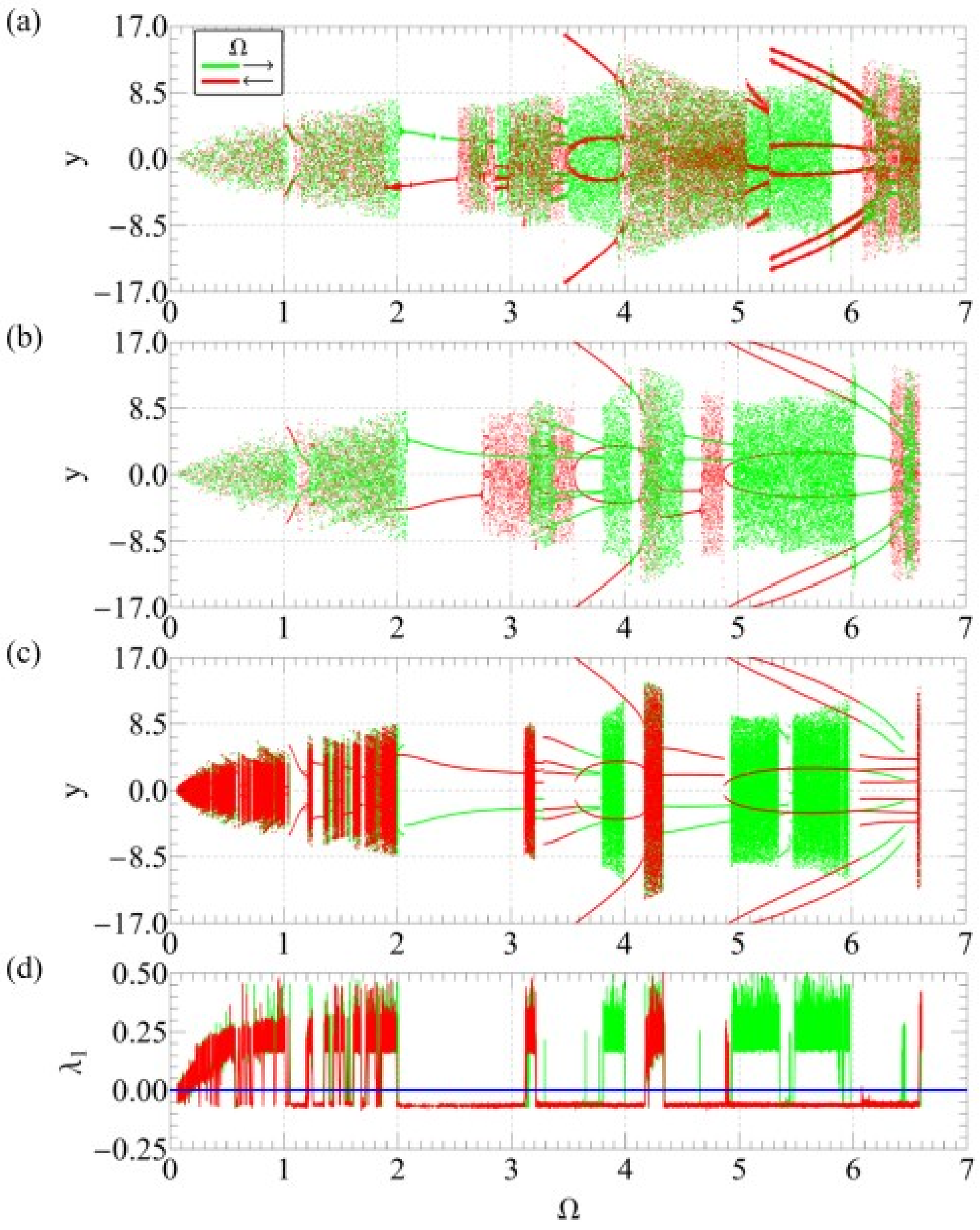
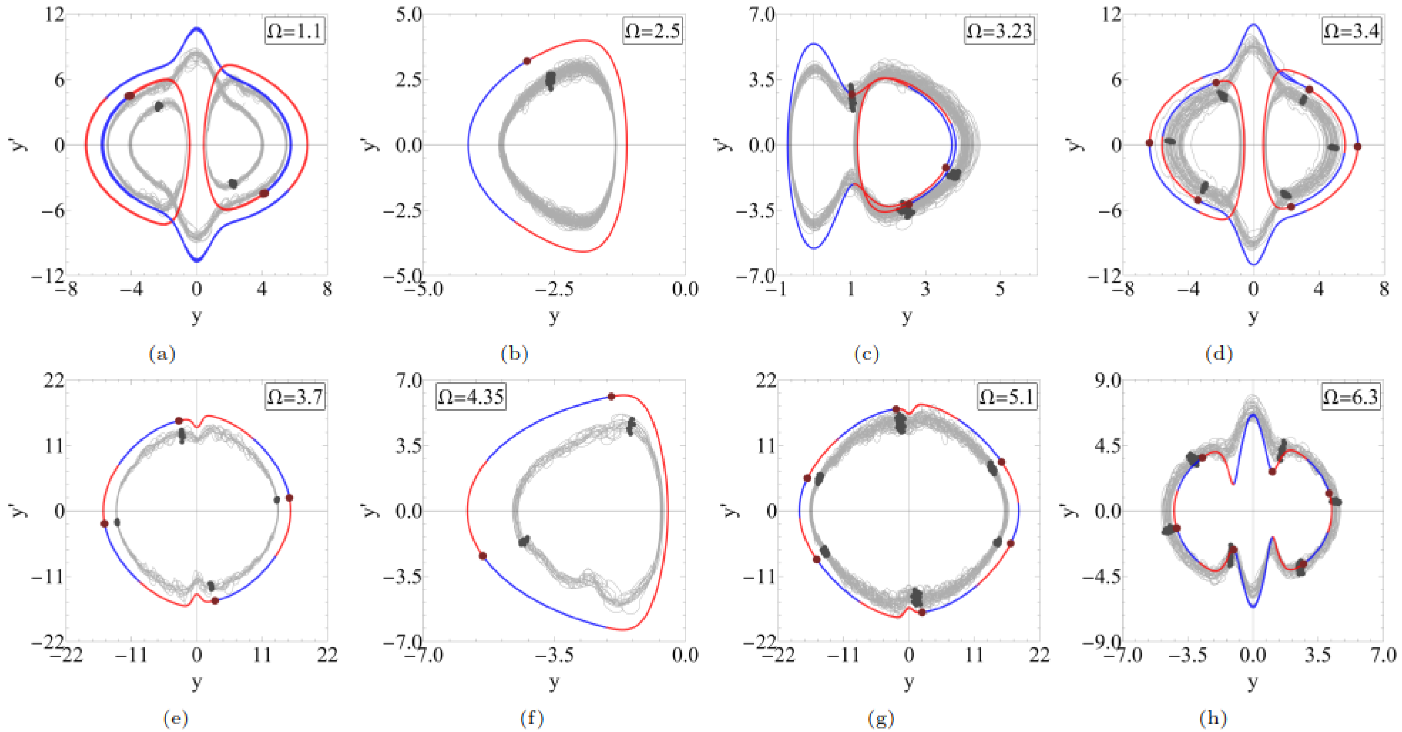
Validation of the model





experiment
simulation

Figure 6: Comparison of simulation and experimental phase trajectories with Poincaré sections.

Figure 5: Bifurcation diagrams obtained for increasing/decreasing Ω: (a) experiment, (b) simulation imitating experiment, (c) classic simulation and (d) the largest Lyapunov exponent λ1 corresponding to the classic simulation.

System under excitation with a controlled phase

$$y'' + \beta y' + \alpha y + \gamma \sin\left(\frac{1}{\gamma} y\right) + \left[\delta + \zeta \exp\left(\nu y'^2\right)\right] \tanh(\sigma y') = A_0 \exp(-y^2) \sin(\Omega x + \phi_0) \quad (7)$$

$$\phi_0(y) = py \quad (8)$$

System of the magnetic pendulum under excitation with controlled phase

$$y'' + \beta y' + \alpha y + \gamma \sin\left(\frac{1}{\gamma} y\right) + \left[\delta + \zeta \exp\left(\nu y'^2\right)\right] \tanh(\sigma y') = A_0 \exp(-y^2) \sin(\Omega x + py) \quad (9)$$

System under excitation with a controlled phase

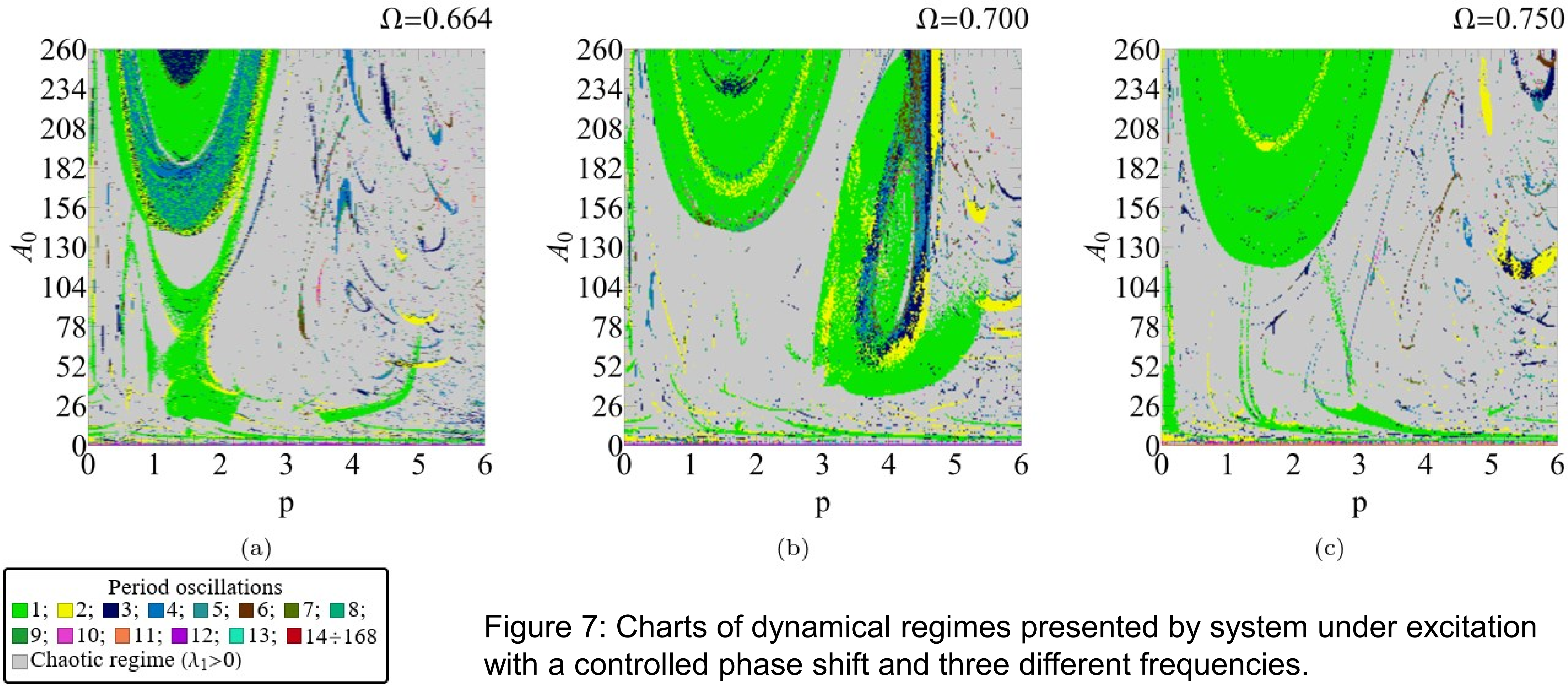
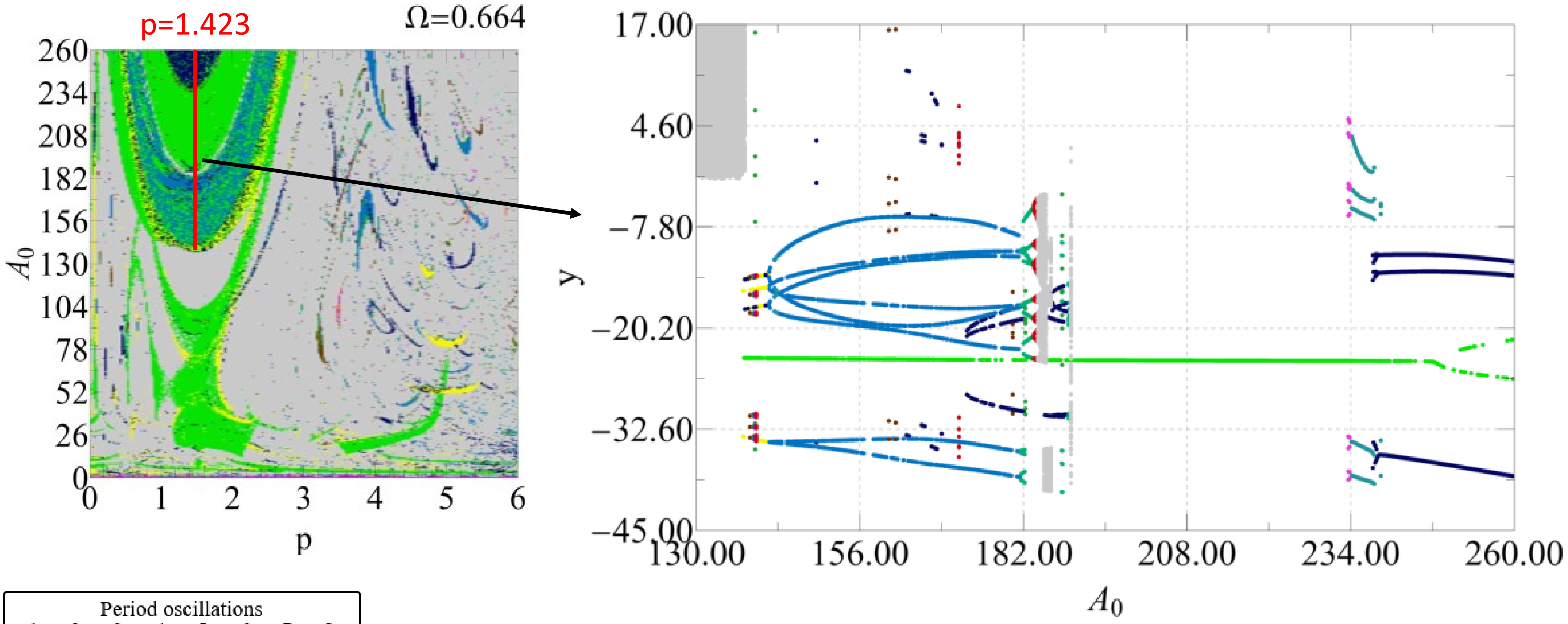
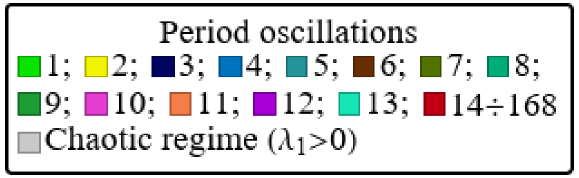


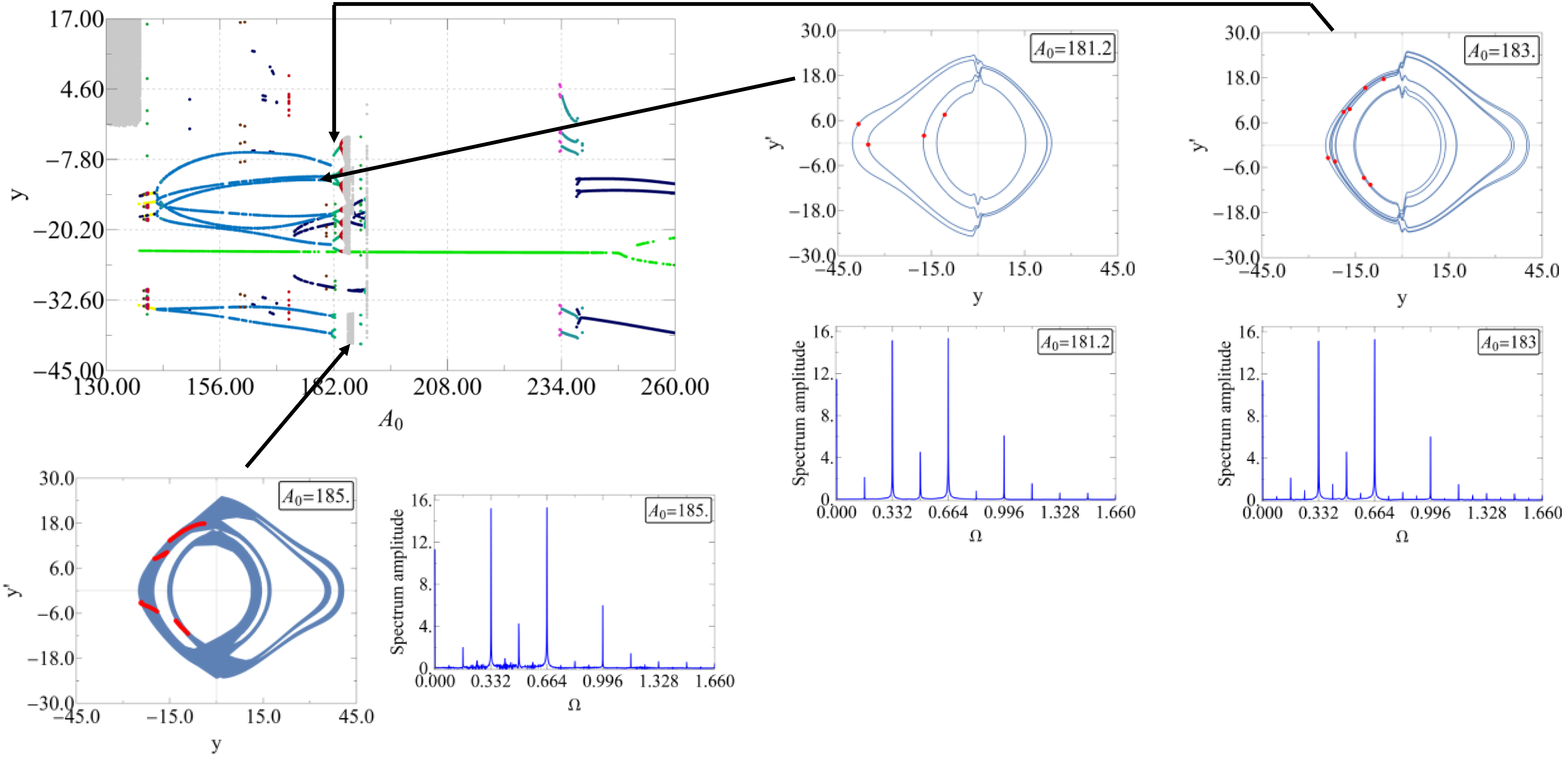
Figure 7: Charts of dynamical regimes presented by system under excitation with a controlled phase shift and three different frequencies.

System under excitation with a controlled phase

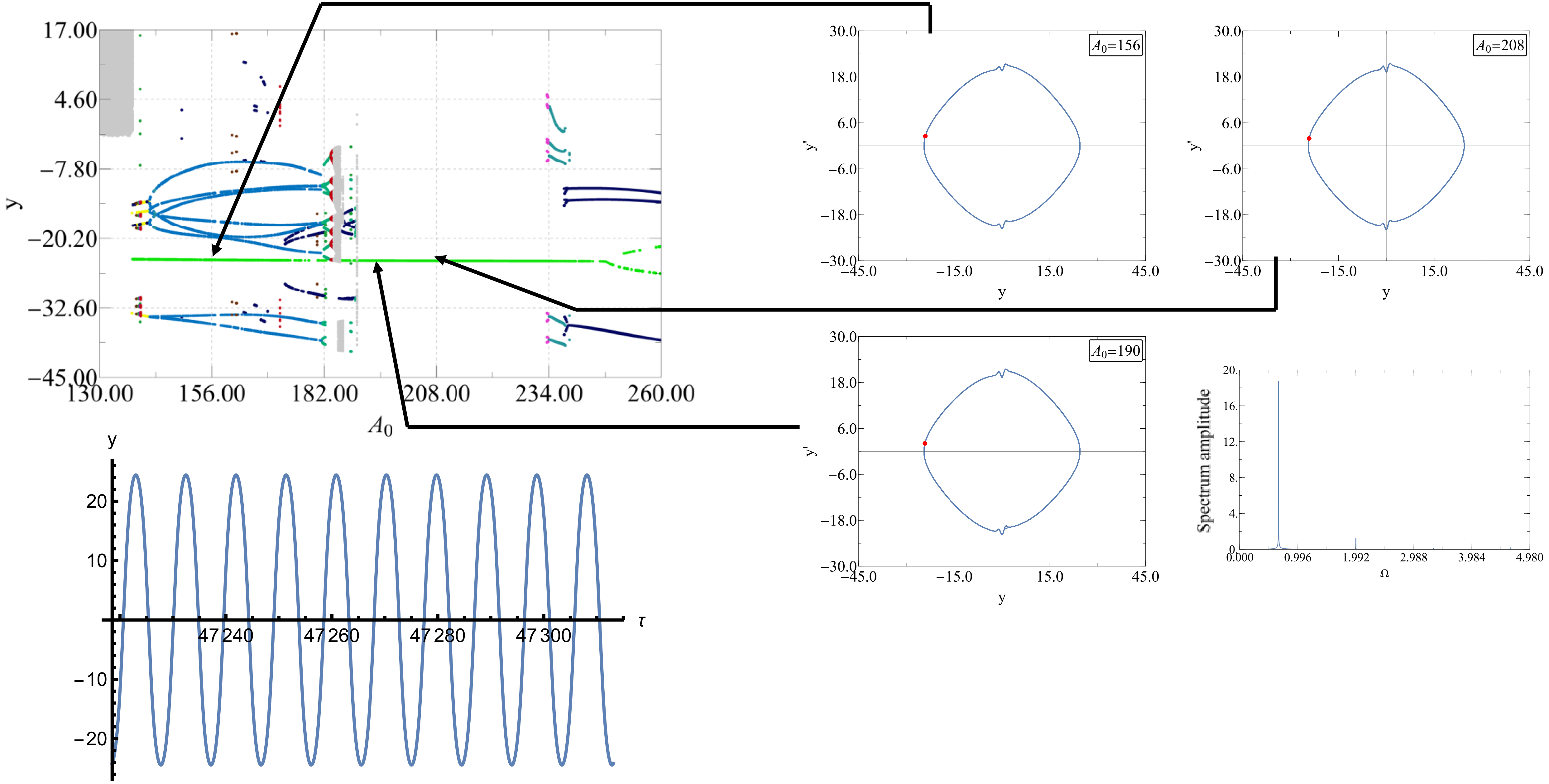




System under excitation with a controlled phase



System under excitation with a controlled phase



Concluding remarks

1. Magnetic interaction introduce parametric excitation to the system, stiffness varying in time.

2. System is multistable and sensitive to initial conditions

3. Periodic modes of the system with a controlled phase are arranged in an elliptical maneuver on a plane of phase slope p and magnetic interaction amplitude A_0.

4. In a phase-controlled system, the value of the magnetic interaction does not always affect the system's trajectory.

Thank you for the attention!

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