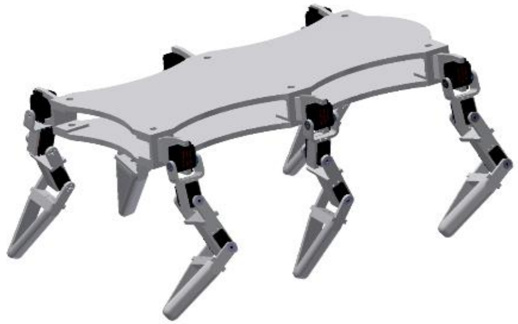
Modelowanie, analiza i wizualizacja maszyn kroczących w środowisku Mathematica

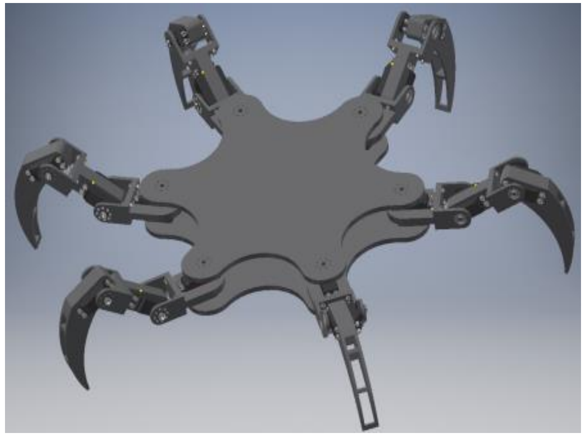
Dr hab. inż. Dariusz Grzelczyk

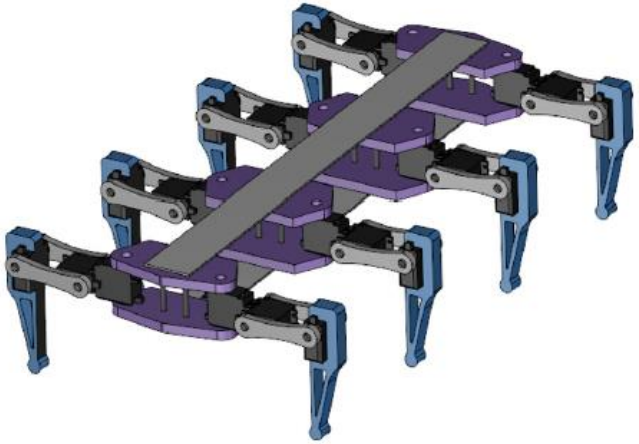
Katedra Automatyki, Biomechaniki i Mechatroniki

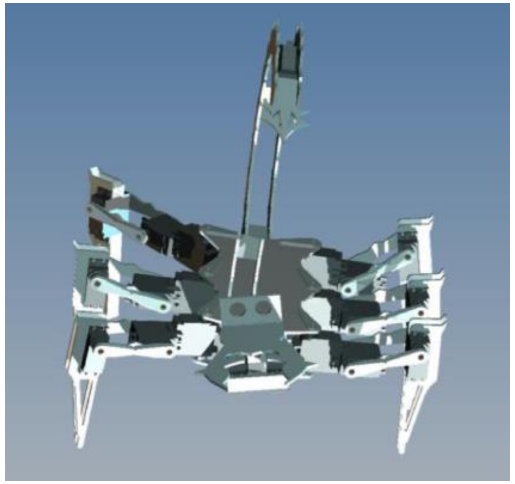
- Modelowanie matematyczne kinematyki i dynamiki maszyn kroczących wzorowanych biologicznie na owadach i ssakach o różnej liczbie kończyn.
- Przedstawienie różnych wzorców chodu i zbadanie ich wpływu na kinematykę, dynamikę i stabilność robota kroczącego w trakcie chodu.
- Opracowanie i sterowanie konstrukcji maszyny kroczącej jako stacjonarnej lub mobilnej platformy Stewarta oraz jej stabilizacja na drgającym i niestabilnym podłożu.

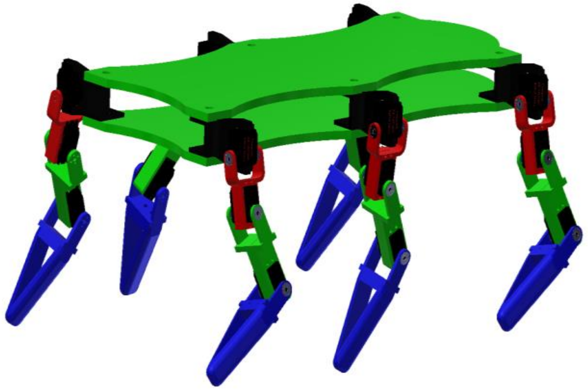
Modele CAD

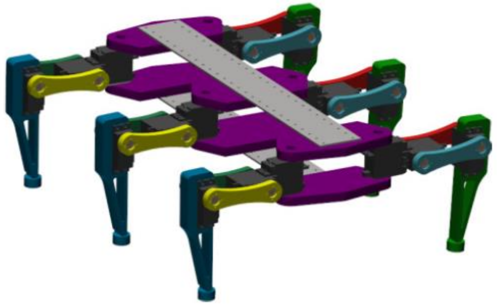




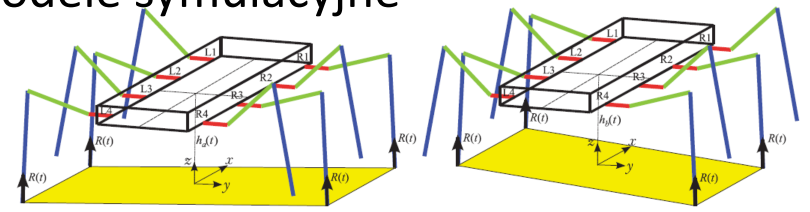


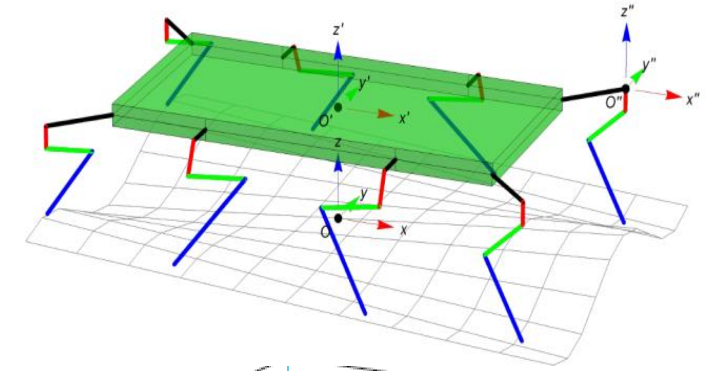


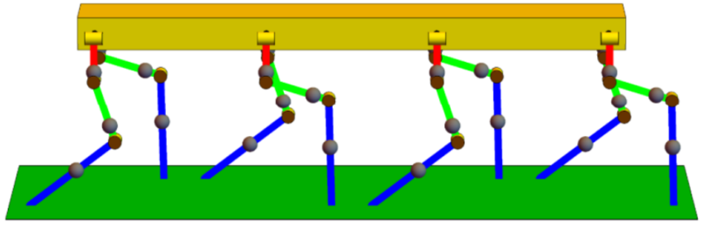


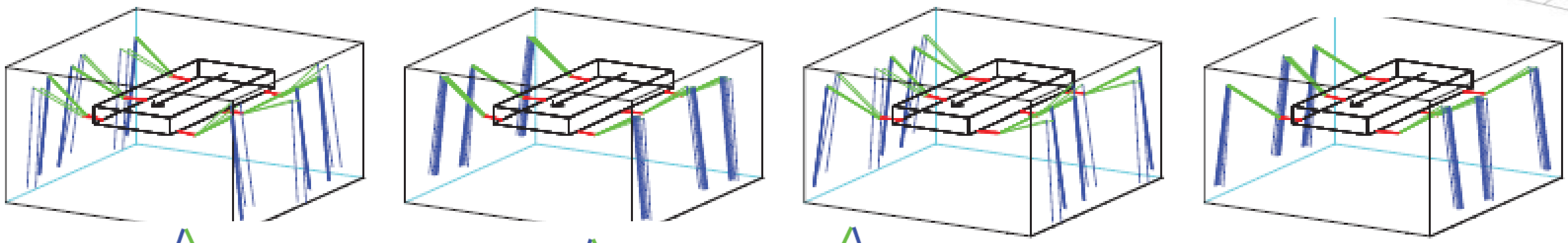


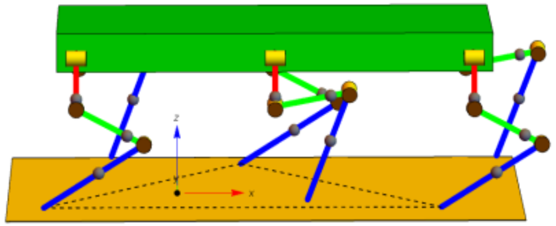
Modele symulacyjne

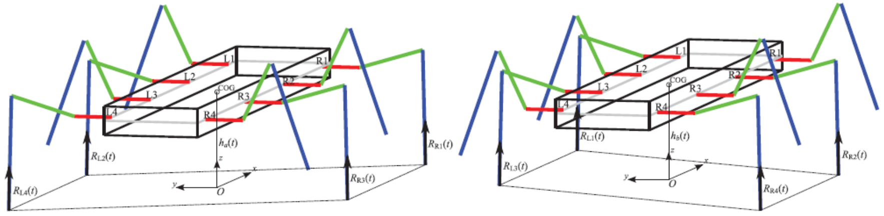


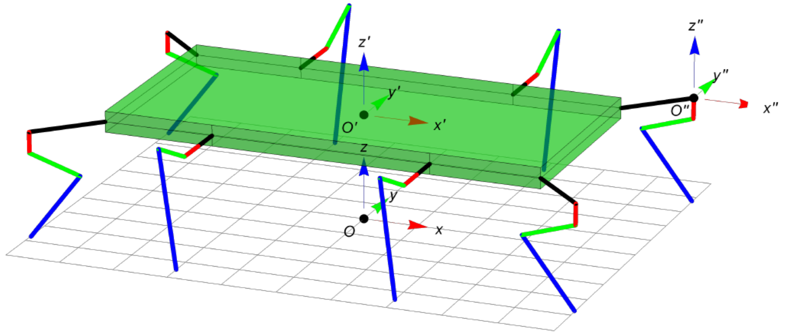


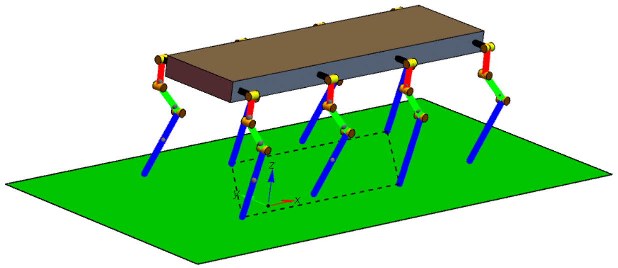


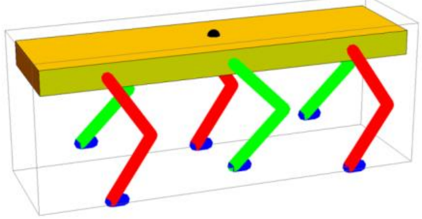


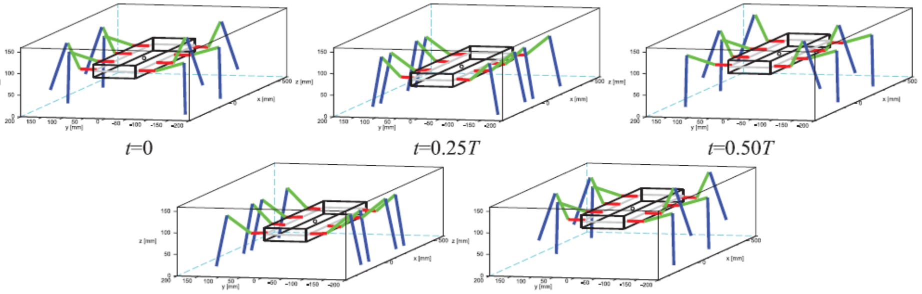




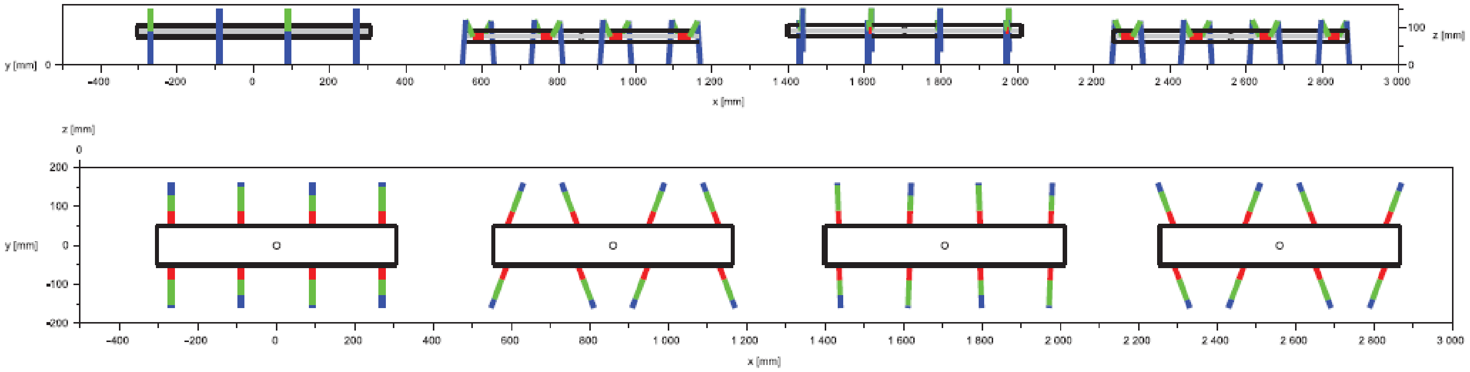


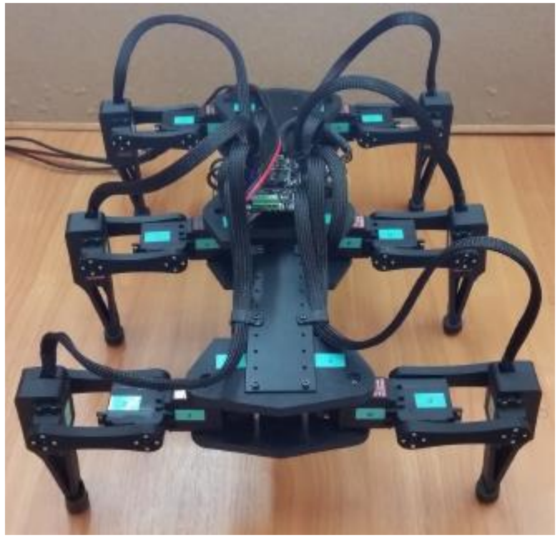


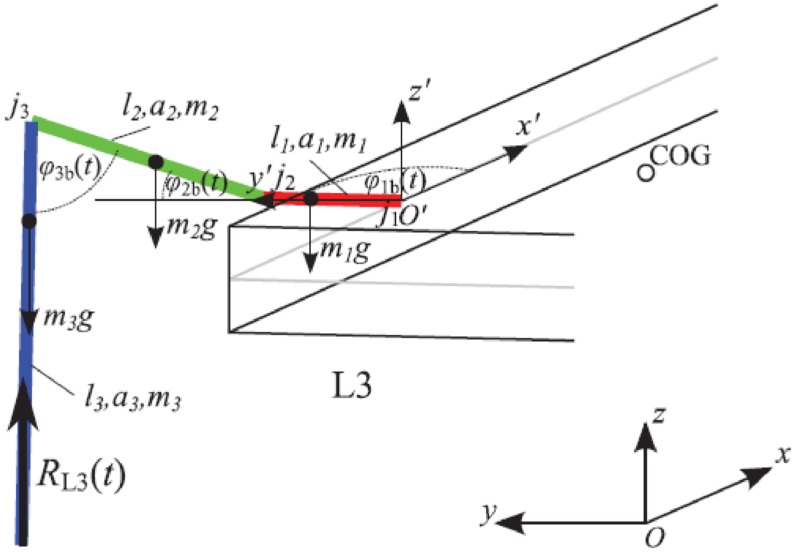




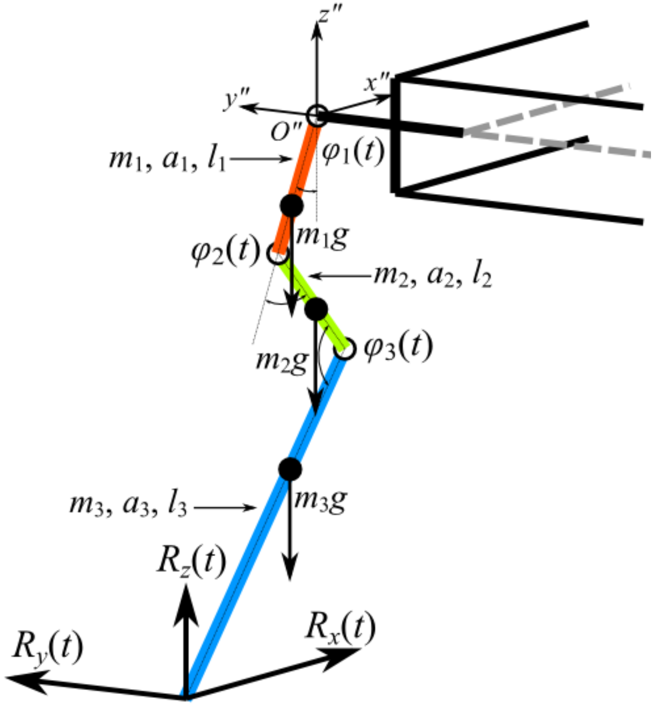
t=0 t=0.25T t=0.50T t=0.75T t=1.00T

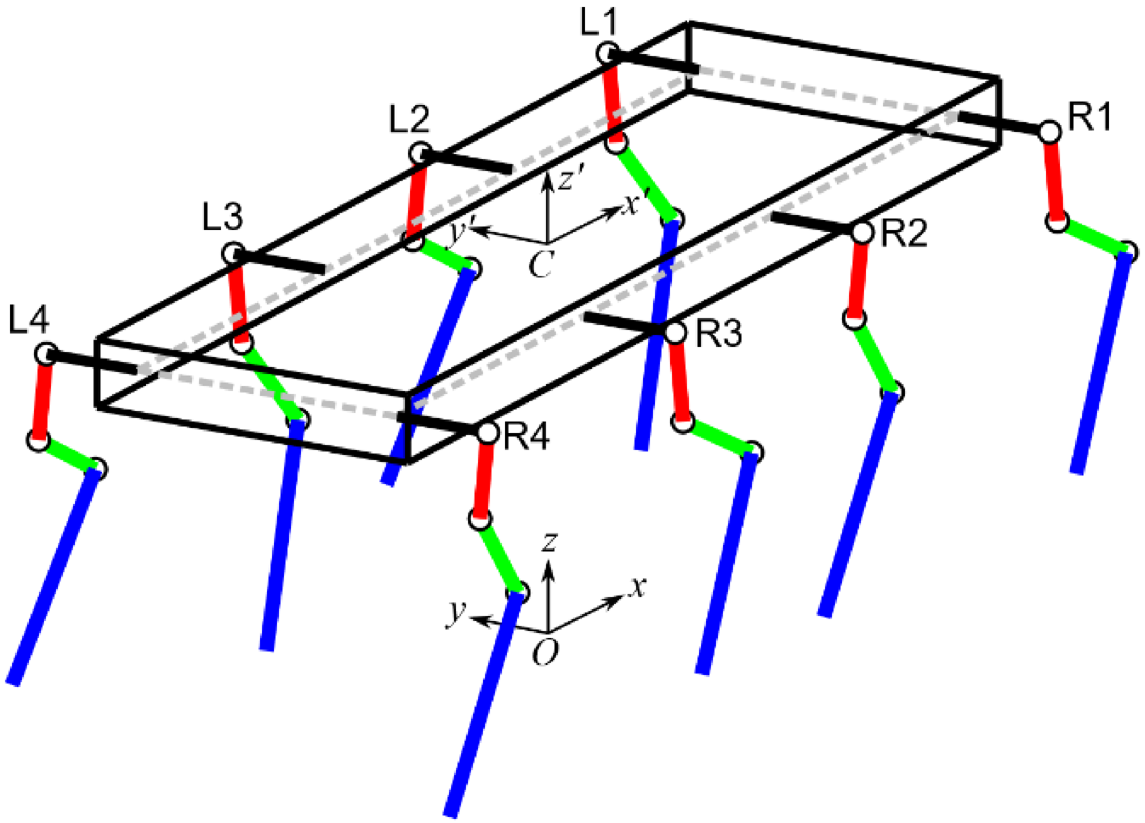


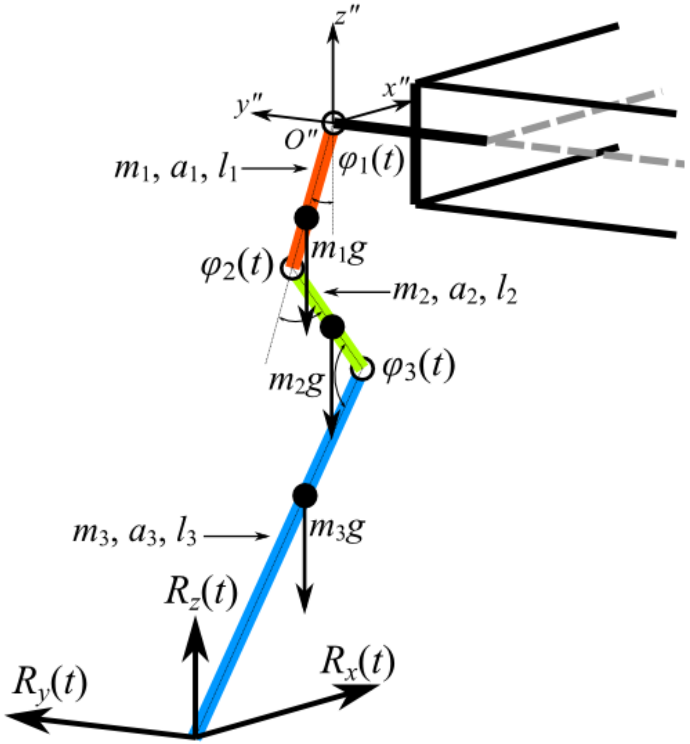



j_3
l_2, a_2, m_2
$\varphi_{3b}(t)$
$\varphi_{2b}(t)$
y' j_2
l_1, a_1, m_1
z'
x'
COG
$\varphi_{1b}(t)$
$j_1 O'$
$m_2 g$
$m_1 g$
$m_3 g$
l_3, a_3, m_3
L3
$R_{L3}(t)$
z
x
y
O

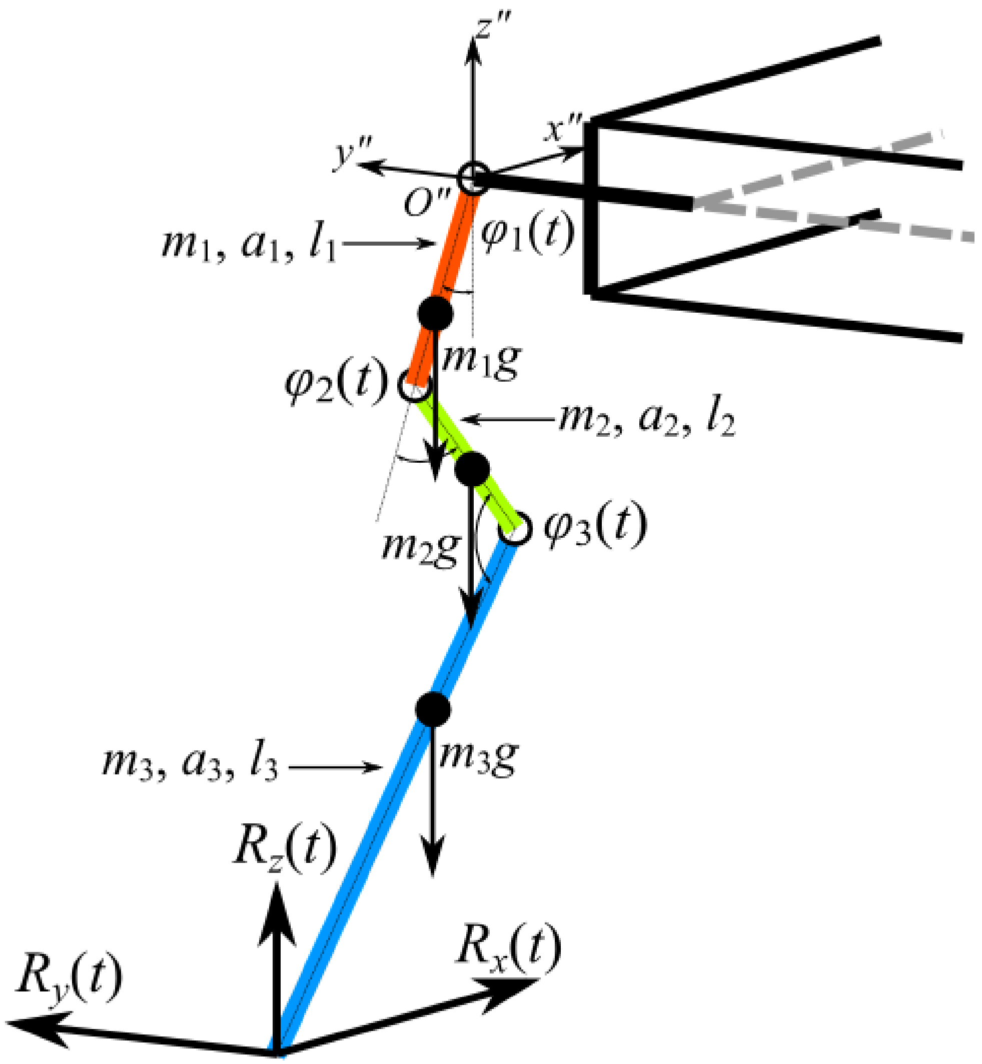

Unitree


z''
x''
y''
O''
m_1, a_1, l_1
$\varphi_1(t)$
$m_1 g$
$\varphi_2(t)$
m_2, a_2, l_2
$\varphi_3(t)$
$m_2 g$
m_3, a_3, l_3
$m_3 g$
$R_z(t)$
$R_y(t)$
$R_x(t)$





Parameters	Symbol	Unit	Value
Mass of the trunk (without legs)	M	[kg]	1.0
Masses of legs' links (with servos)	m_1; m_2; m_3	[kg]	0.111; 0.140; 0.124
Lengths of legs' links	l_1; l_2; l_3	[m]	0.065; 0.100; 0.165
Positions of links' mass centers	a_1; a_2; a_3	[m]	0.052; 0.075; 0.075
Distances between the legs	L; H	[m]	0.263; 0.210

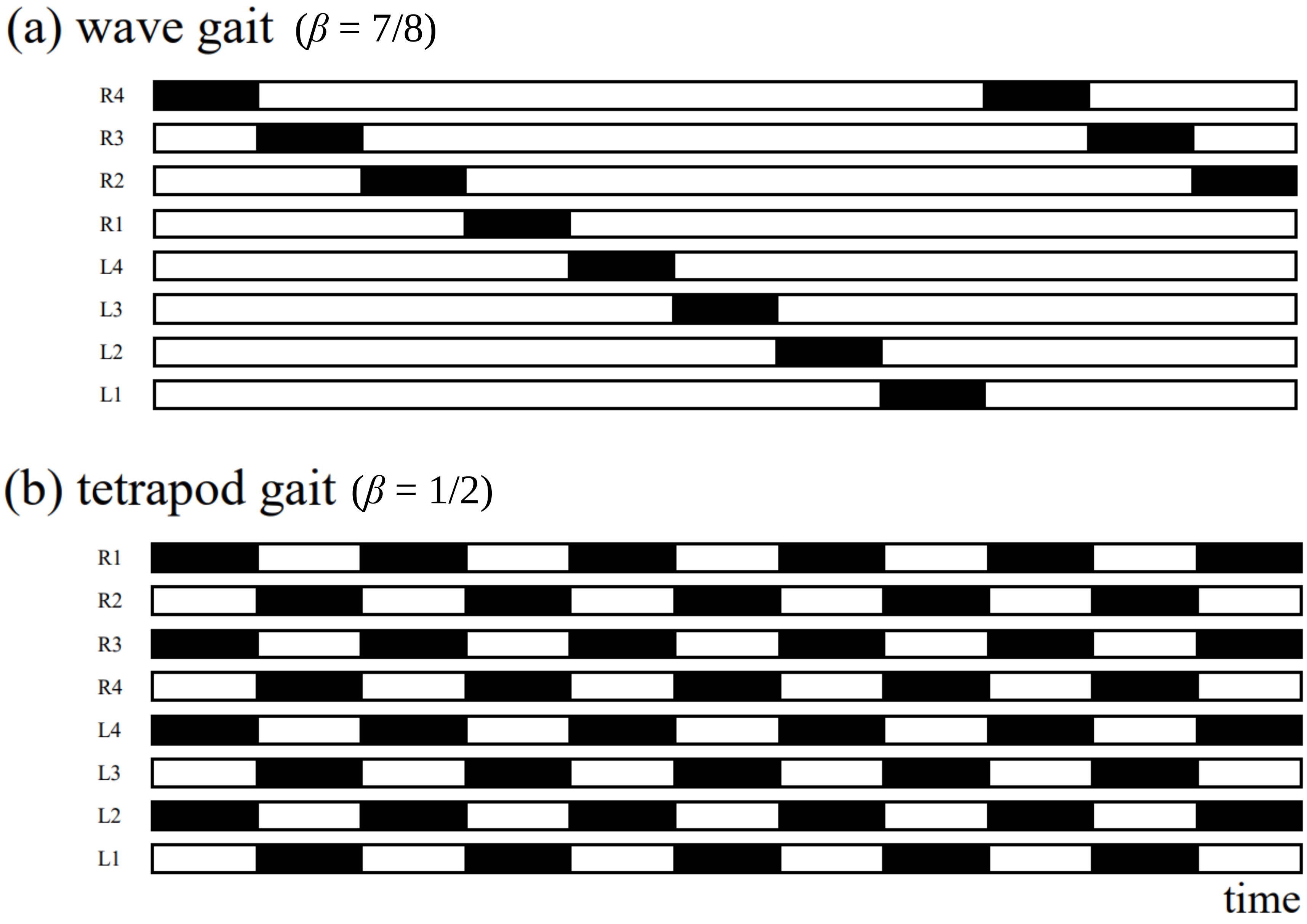


$$\begin{cases} x''(t) = l_2 \sin\varphi_2(t) - l_3 \sin\left(\pi - \varphi_2(t) - \varphi_3(t)\right), \\ y''(t) = \left(-l_1 - l_2\cos\varphi_2(t) - l_3\cos\left(\pi - \varphi_2(t) - \varphi_3(t)\right)\right)\sin\varphi_1(t), \\ z''(t) = \left(-l_1 - l_2\cos\varphi_2(t) - l_3\cos\left(\pi - \varphi_2(t) - \varphi_3(t)\right)\right)\cos\varphi_1(t), \end{cases}$$

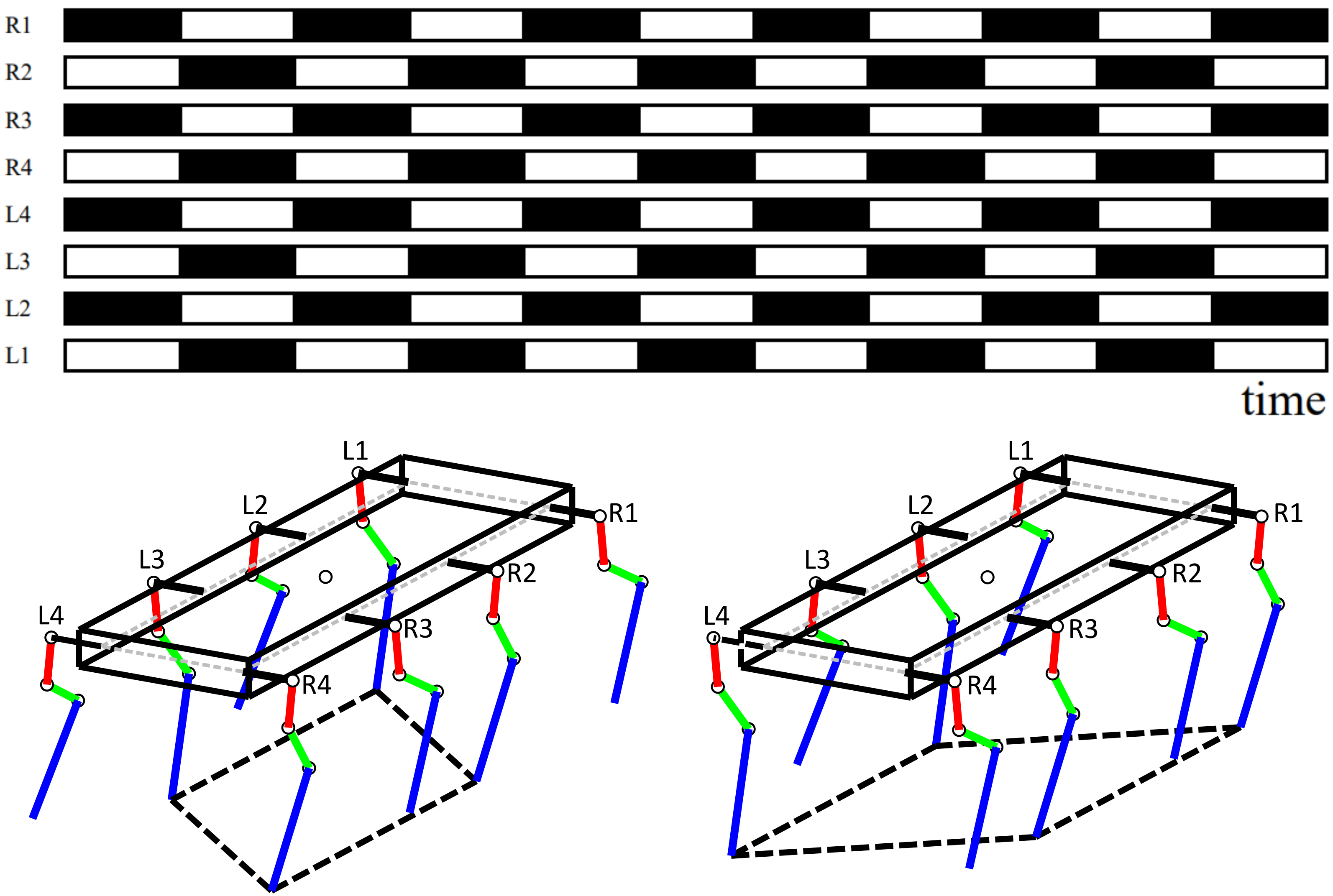
$$\begin{cases} \varphi_1(t) = \operatorname{atan}\left(\dfrac{y''(t)}{-z''(t)}\right), \\ \varphi_2(t) = \operatorname{atan}\left(\dfrac{x''(t)}{d_1(t)}\right) + \operatorname{acos}\left(\dfrac{l_2^2 + [d_2(t)]^2 - l_3^2}{2l_2d_2(t)}\right), \\ \varphi_3(t) = \operatorname{acos}\left(\dfrac{l_2^2 + l_3^2 - [d_2(t)]^2}{2l_2l_3}\right), \end{cases}$$

$$d_1(t) = \sqrt{[y''(t)]^2 + [z''(t)]^2} - l_1$$

$$d_2(t) = \sqrt{[d_1(t)]^2 + [x''(t)]^2}$$


(a) wave gait (β = 7/8)
R4
R3
R2
R1
L4
L3
L2
L1
(b) tetrapod gait (β = 1/2)
R1
R2
R3
R4
L4
L3
L2
L1
time

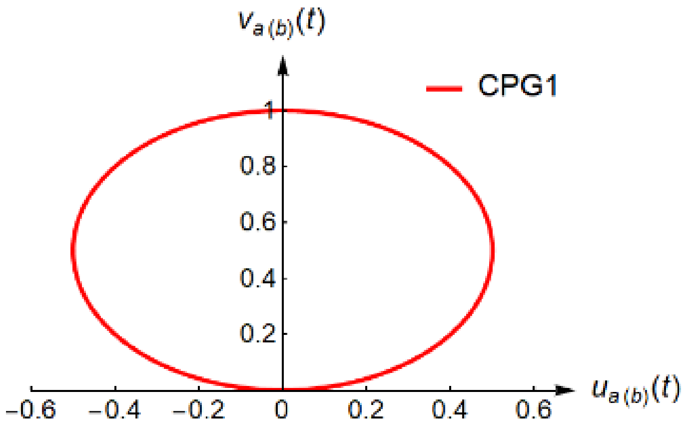
(b) tetrapod gait ($\beta = 1/2$)



Wzorce chodu

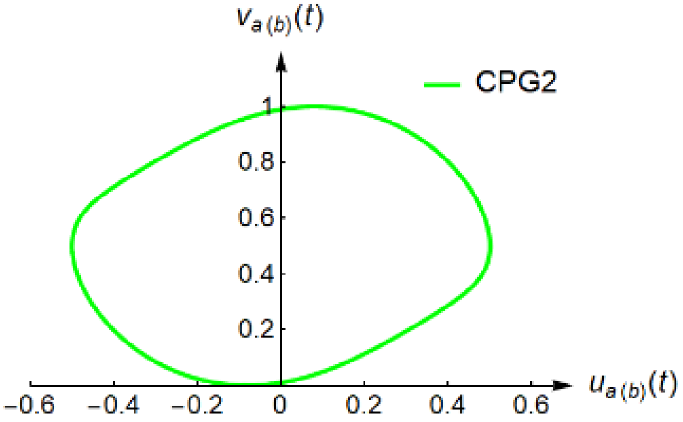
CPG1: Two uncoupled Hopf oscillators

$$\begin{cases} \dot{u}_a(t) = \left(\sigma - u_a^2(t) - v_a^2(t)\right)u_a(t) + \omega v_a(t), \\ \dot{v}_a(t) = \left(\sigma - u_a^2(t) - v_a^2(t)\right)v_a(t) - \omega u_a(t), \\ \dot{u}_b(t) = \left(\sigma - u_b^2(t) - v_b^2(t)\right)u_b(t) + \omega v_b(t), \\ \dot{v}_b(t) = \left(\sigma - u_b^2(t) - v_b^2(t)\right)v_b(t) - \omega u_b(t), \end{cases}$$



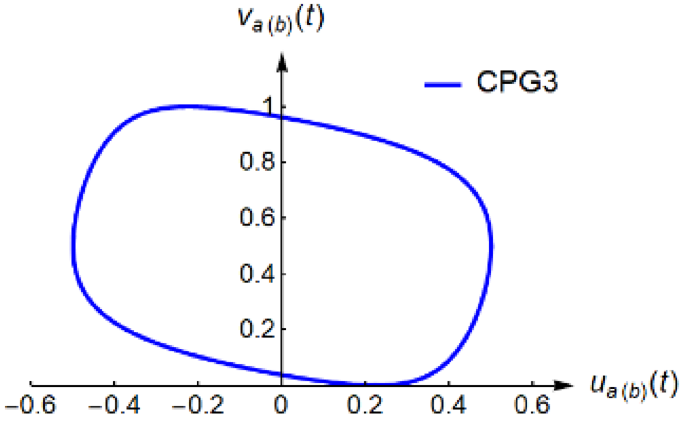
CPG2: Two uncoupled hybrid van der Pol-Rayleigh oscillators

$$\begin{cases} \dot{u}_a(t) = v_a(t), \\ \dot{v}_a(t) = \varepsilon\left(1 - u_a^2(t)\right)v_a(t) + \delta\left(1 - v_a^2(t)\right)v_a(t) - \omega^2 u_a(t), \\ \dot{u}_b(t) = v_b(t), \\ \dot{v}_b(t) = \varepsilon\left(1 - u_b^2(t)\right)v_b(t) + \delta\left(1 - v_b^2(t)\right)v_b(t) - \omega^2 u_b(t), \end{cases}$$



CPG3: The Toda-Rayleigh lattice

$$\begin{cases} \dot{u}_a(t) = v_a(t), \\ \dot{v}_a(t) = \omega^2\left(e^{u_b(t)-u_a(t)} - e^{u_a(t)-u_b(t)}\right) + \eta\left(1 - v_a^2(t)\right)v_a(t), \\ \dot{u}_b(t) = v_b(t), \\ \dot{v}_b(t) = \omega^2\left(e^{u_a(t)-u_b(t)} - e^{u_b(t)-u_a(t)}\right) + \eta\left(1 - v_b^2(t)\right)v_b(t), \end{cases}$$



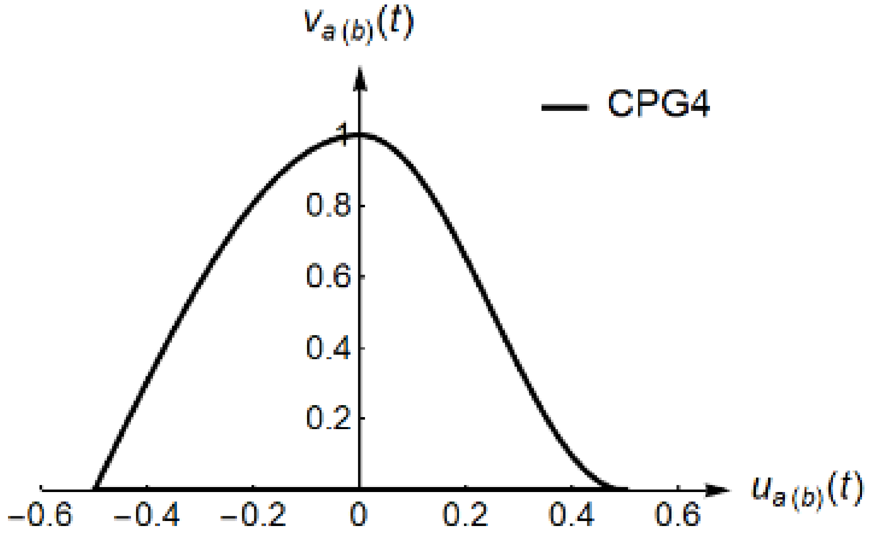
Zaproponowany wzorzec chodu

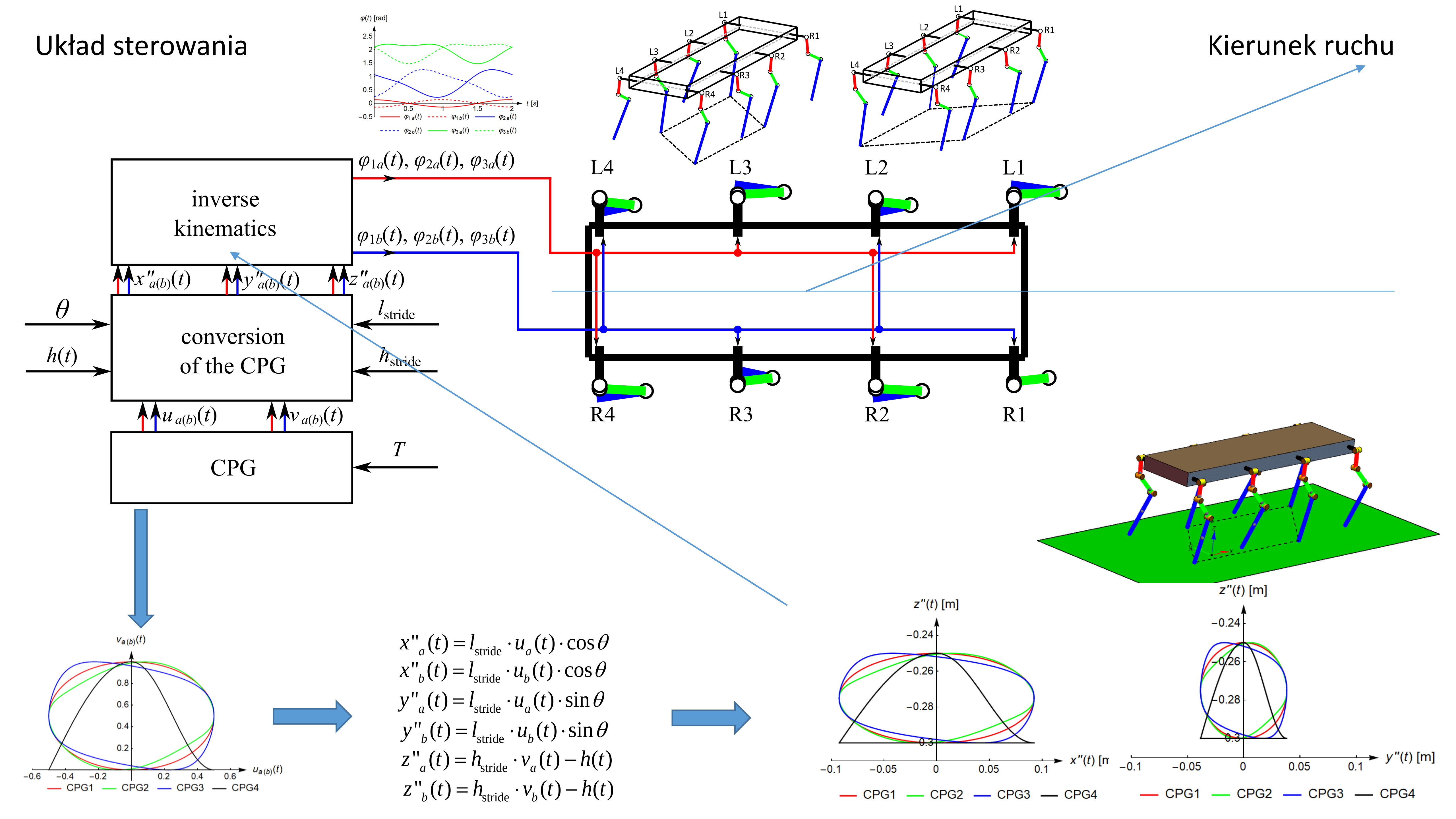
$$\begin{cases} u_a(t) = f_x\left(\mathrm{mod}[t,T]\right), \\ v_a(t) = f_{za}\left(\mathrm{mod}[t,T]\right), \\ u_b(t) = -u_1(t), \\ v_b(t) = f_{zb}\left(\mathrm{mod}[t,T]\right), \end{cases}$$

$$f_x(t) = \begin{cases} \frac{2}{T}t & \text{if} \quad t \in [0,\, 0.25T], \\ \frac{1}{2} - \frac{2}{T}(t - 0.25T) & \text{if} \quad t \in (0.25T,\, 0.75T], \\ -\frac{1}{2} + \frac{2}{T}(t - 0.75T) & \text{if} \quad (0.75T,\, T], \end{cases}$$

$$f_{za}(t) = \begin{cases} \sin^2\left(\frac{2\pi}{T}(t + 0.25T)\right) & \text{if} \quad t \in [0,\, 0.25T], \\ 0 & \text{if} \quad t \in (0.25T,\, 0.75T], \\ -\sin\left(\frac{2\pi}{T}(t + 0.75T)\right) & \text{if} \quad (0.75T,\, T], \end{cases}$$

$$f_{zb}(t) = f_{za}(\mathrm{mod}[t - 0.5T, T])$$



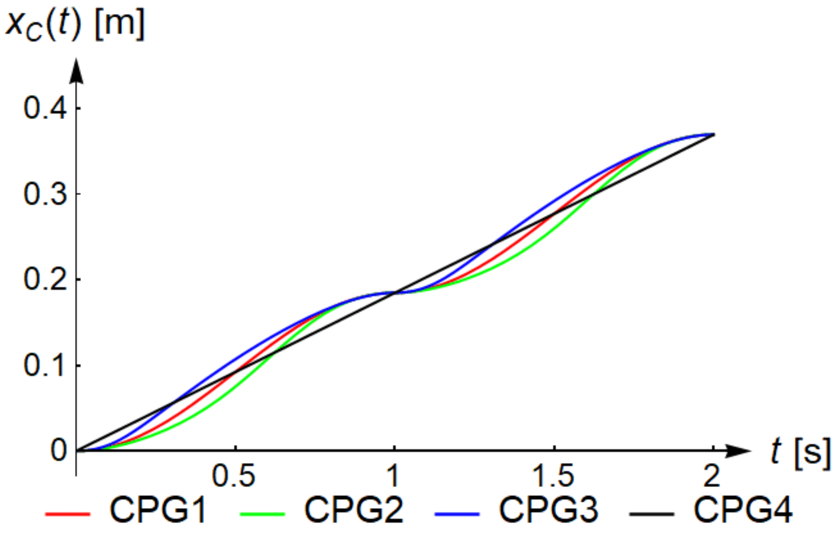

Układ sterowania
Kierunek ruchu
inverse kinematics
conversion of the CPG
CPG
φ1a(t), φ2a(t), φ3a(t)
φ1b(t), φ2b(t), φ3b(t)
x''a(b)(t)
y''a(b)(t)
z''a(b)(t)
θ
h(t)
lstride
hstride
ua(b)(t)
va(b)(t)
T
L4
L3
L2
L1
R4
R3
R2
R1
φ(t) [rad]
t [s]
φ1a(t)
φ1b(t)
φ2a(t)
φ2b(t)
φ3a(t)
φ3b(t)
va(b)(t)
ua(b)(t)
CPG1
CPG2
CPG3
CPG4
x''a(t) = lstride · ua(t) · cos θ
x''b(t) = lstride · ub(t) · cos θ
y''a(t) = lstride · ua(t) · sin θ
y''b(t) = lstride · ub(t) · sin θ
z''a(t) = hstride · va(t) − h(t)
z''b(t) = hstride · vb(t) − h(t)
z''(t) [m]
x''(t) [m]
y''(t) [m]

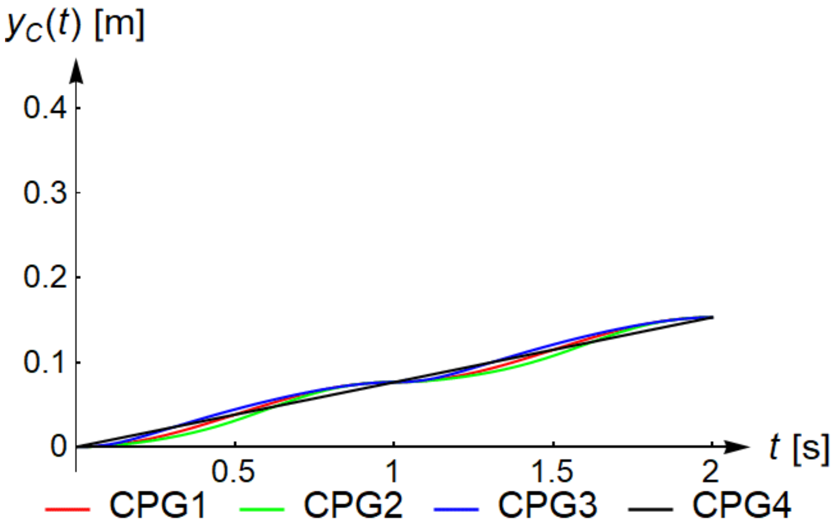
Kinematyka robota

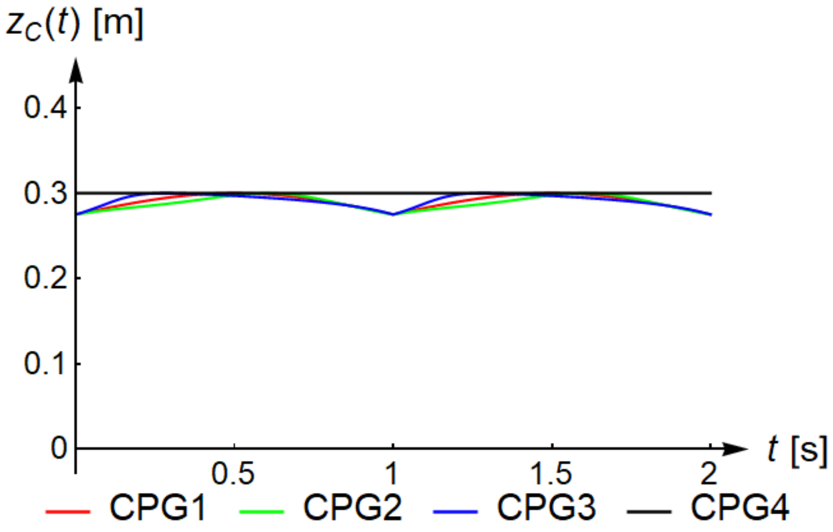
$$x_C(t) = \int_0^t v_{xC}(\tau)d\tau \qquad y_C(t) = \int_0^t v_{yC}(\tau)d\tau \qquad z_C(t) = \begin{cases} -z''_a(t) & \text{if} \quad |z''_a(t)| \geq |z''_b(t)|, \\ -z''_b(t) & \text{if} \quad |z''_a(t)| < |z''_b(t)|, \end{cases}$$

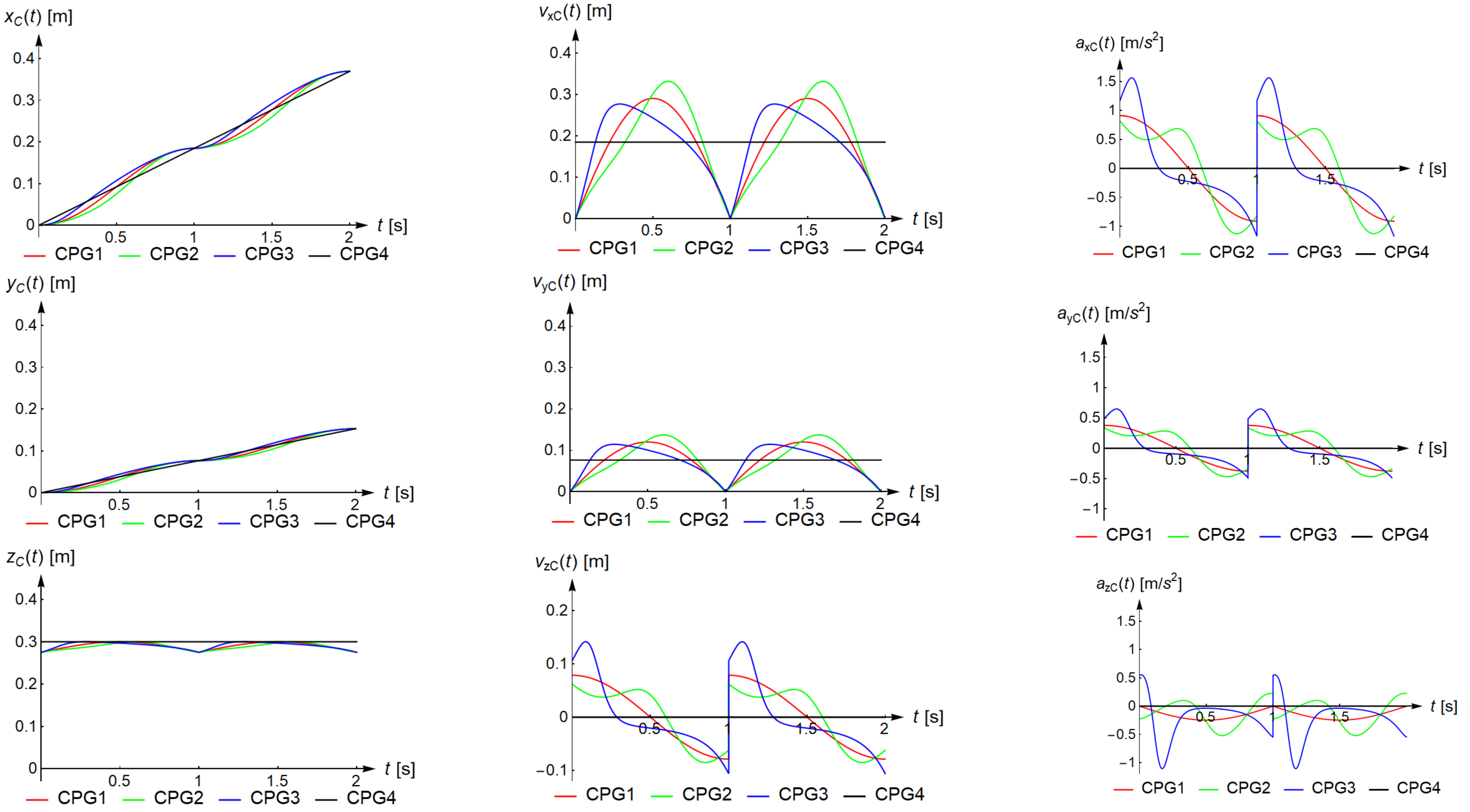
gdzie:

$$v_{xC}(t) = \begin{cases} -\dot{x}''_a(t) & \text{if} \quad |z''_a(t)| \geq |z''_b(t)|, \\ -\dot{x}''_b(t) & \text{if} \quad |z''_a(t)| < |z''_b(t)|, \end{cases} \qquad v_{yC}(t) = \begin{cases} -\dot{y}''_a(t) & \text{if} \quad |z''_a(t)| \geq |z''_b(t)|, \\ -\dot{y}''_b(t) & \text{if} \quad |z''_a(t)| < |z''_b(t)|. \end{cases}$$









x_C(t) [m]
t [s]
CPG1 CPG2 CPG3 CPG4
v_xC(t) [m]
t [s]
CPG1 CPG2 CPG3 CPG4
a_xC(t) [m/s^2]
t [s]
CPG1 CPG2 CPG3 CPG4
y_C(t) [m]
t [s]
CPG1 CPG2 CPG3 CPG4
v_yC(t) [m]
t [s]
CPG1 CPG2 CPG3 CPG4
a_yC(t) [m/s^2]
t [s]
CPG1 CPG2 CPG3 CPG4
z_C(t) [m]
t [s]
CPG1 CPG2 CPG3 CPG4
v_zC(t) [m]
t [s]
CPG1 CPG2 CPG3 CPG4
a_zC(t) [m/s^2]
t [s]
CPG1 CPG2 CPG3 CPG4

COM:

$$x_{COM}(t)=\frac{\left(M+M_L\right)x_{\mathrm{C}}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i x_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i x_i^{(\mathrm{R}j)}(t)}{M+M_L+8\sum_{i=1}^{3}m_i},$$

$$y_{COM}(t)=\frac{\left(M+M_L\right)y_{\mathrm{C}}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i y_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i y_i^{(\mathrm{R}j)}(t)}{M+M_L+8\sum_{i=1}^{3}m_i},$$

$$z_{COM}(t)=\frac{\left(M+M_L\right)z_{\mathrm{C}}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i z_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i z_i^{(\mathrm{R}j)}(t)}{M+M_L+8\sum_{i=1}^{3}m_i}.$$

ZMP:

$$x_{ZMP}(t) = \frac{\Sigma_A(t) - \Sigma_B(t)}{\Sigma_E(t)} \qquad y_{ZMP}(t) = \frac{\Sigma_C(t) - \Sigma_D(t)}{\Sigma_E(t)}$$

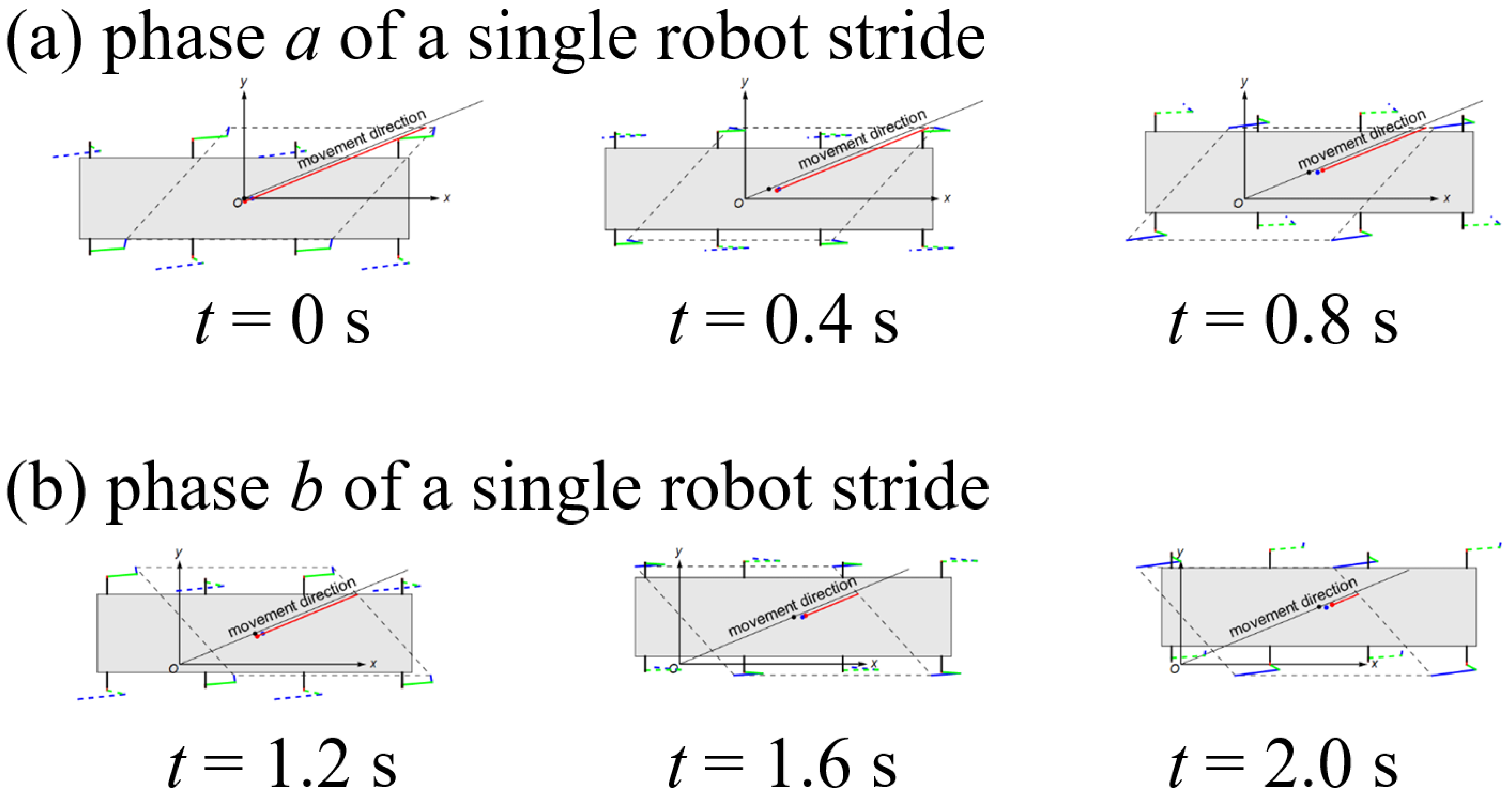
$$\Sigma_A(t) = (M + M_L)(\ddot{z}_C(t) + g_z)x_C(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{z}_i^{(Lj)}(t) + g_z\right)x_i^{(Lj)}(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{z}_i^{(Rj)}(t) + g_z\right)x_i^{(Rj)}(t),$$

$$\Sigma_B(t) = (M + M_L)(\ddot{x}_C(t) + g_x)z_C(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{x}_i^{(Lj)}(t) + g_x\right)z_i^{(Lj)}(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{x}_i^{(Rj)}(t) + g_x\right)z_i^{(Rj)}(t),$$

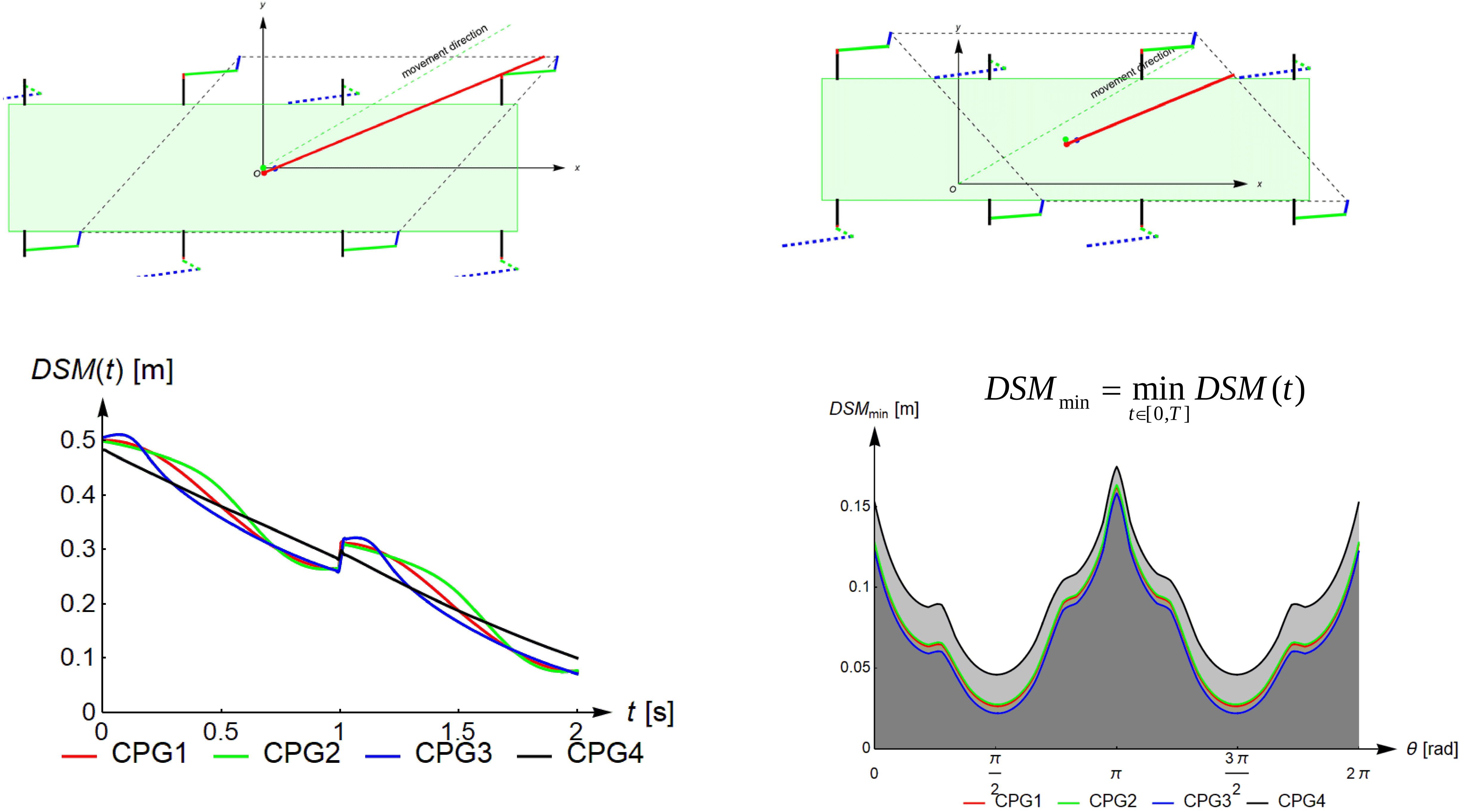
$$\Sigma_C(t) = (M + M_L)(\ddot{z}_C(t) + g_z)y_C(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{z}_i^{(Lj)}(t) + g_z\right)y_i^{(Lj)}(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{z}_i^{(Rj)}(t) + g_z\right)y_i^{(Rj)}(t),$$

$$\Sigma_D(t) = (M + M_L)(\ddot{y}_C(t) + g_y)z_C(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{y}_i^{(Lj)}(t) + g_y\right)z_i^{(Lj)}(t) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{y}_i^{(Rj)}(t) + g_y\right)z_i^{(Rj)}(t),$$

$$\Sigma_E(t) = (M + M_L)(\ddot{z}_C(t) + g_z) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{z}_i^{(Lj)}(t) + g_z\right) + \sum_{j=1}^{4}\sum_{i=1}^{3} m_i\left(\ddot{z}_i^{(Rj)}(t) + g_z\right).$$



(a) phase a of a single robot stride
y
x
O
movement direction
t = 0 s
t = 0.4 s
t = 0.8 s
(b) phase b of a single robot stride
t = 1.2 s
t = 1.6 s
t = 2.0 s



movement direction
movement direction
y
x
O
DSM(t) [m]
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
t [s]
CPG1
CPG2
CPG3
CPG4
$DSM_{\min} = \min_{t\in[0,T]} DSM(t)$
$DSM_{\min}$ [m]
0.15
0.1
0.05
0
0
π/2
π
3π/2
2π
θ [rad]
CPG1
CPG2
CPG3
CPG4

Siły reakcji podłoża:

$$\left(M+M_L\right)\ddot{\mathbf{r}}_C(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{\mathbf{r}}_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{\mathbf{r}}_i^{(\mathrm{R}j)}(t)=$$

$$=\sum_{j=1}^{4}\mathbf{R}^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\mathbf{R}^{(\mathrm{R}j)}(t)+\left(M+M_L+8\sum_{i=1}^{3}m_i\right)\mathbf{g},$$

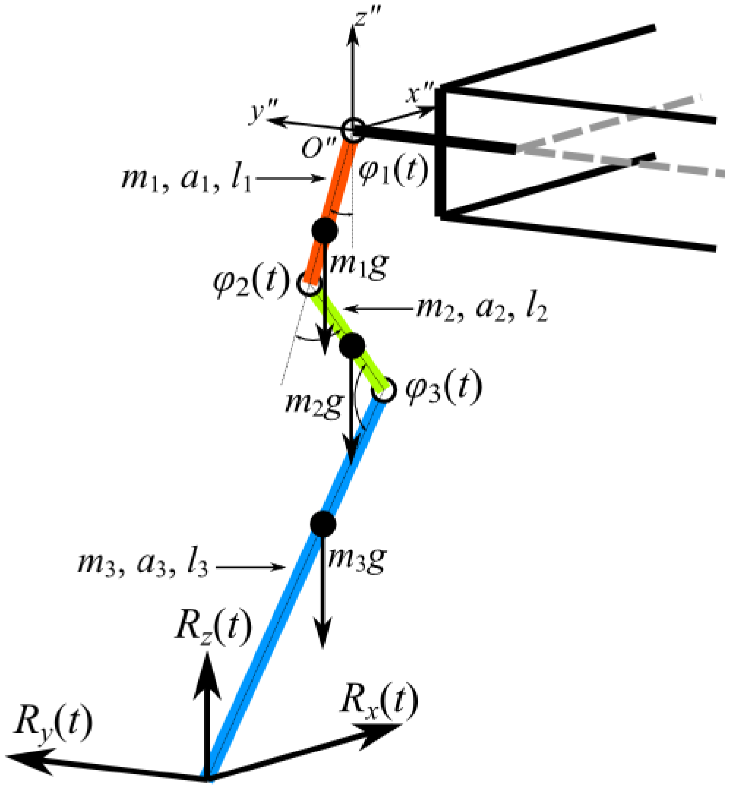
$$\left(M+M_L\right)\ddot{x}_\mathrm{C}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{x}_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{x}_i^{(\mathrm{R}j)}(t)=\sum_{j=1}^{4}R_x^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}R_x^{(\mathrm{R}j)}(t),$$

$$\left(M+M_\mathrm{L}\right)\ddot{y}_\mathrm{C}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{y}_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{y}_i^{(\mathrm{R}j)}(t)=\sum_{j=1}^{4}R_y^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}R_y^{(\mathrm{R}j)}(t),$$

$$\left(M+M_L\right)\ddot{z}_\mathrm{C}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{z}_i^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}\sum_{i=1}^{3}m_i\ddot{z}_i^{(\mathrm{R}j)}(t)+\left(M+M_L+8\sum_{i=1}^{3}m_i\right)g=\sum_{j=1}^{4}R_z^{(\mathrm{L}j)}(t)+\sum_{j=1}^{4}R_z^{(\mathrm{R}j)}(t).$$

Założenia:

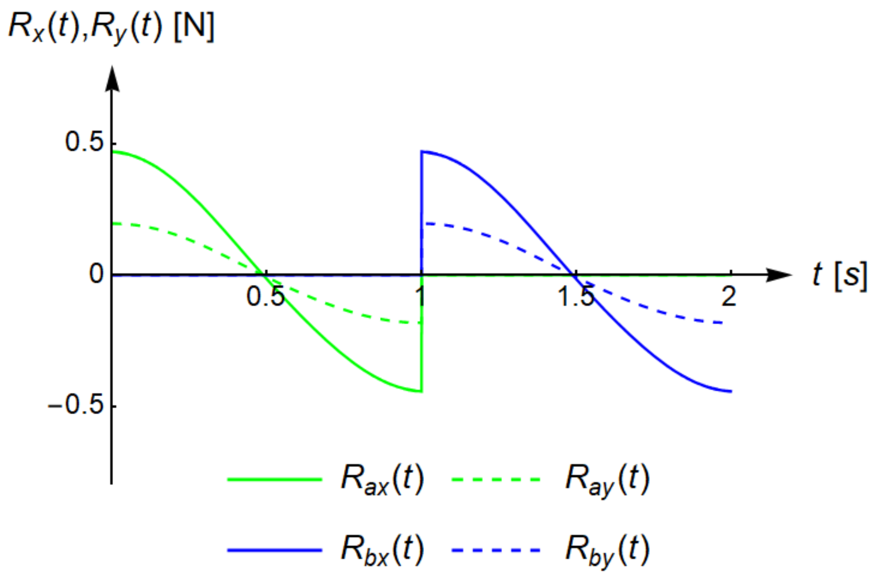
$$R_x^{(L1)}(t) = R_x^{(L3)}(t) = R_x^{(R2)}(t) = R_x^{(R4)}(t) = R_{ax}(t)$$
$$R_x^{(L2)}(t) = R_x^{(L4)}(t) = R_x^{(R1)}(t) = R_x^{(R3)}(t) = R_{bx}(t)$$
$$R_y^{(L1)}(t) = R_y^{(L3)}(t) = R_y^{(R2)}(t) = R_y^{(R4)}(t) = R_{ay}(t)$$
$$R_y^{(L2)}(t) = R_y^{(L4)}(t) = R_y^{(R1)}(t) = R_y^{(R3)}(t) = R_{by}(t)$$
$$R_z^{(L1)}(t) = R_z^{(L3)}(t) = R_z^{(R2)}(t) = R_z^{(R4)}(t) = R_{az}(t)$$
$$R_z^{(L2)}(t) = R_z^{(L4)}(t) = R_z^{(R1)}(t) = R_z^{(R3)}(t) = R_{bz}(t)$$



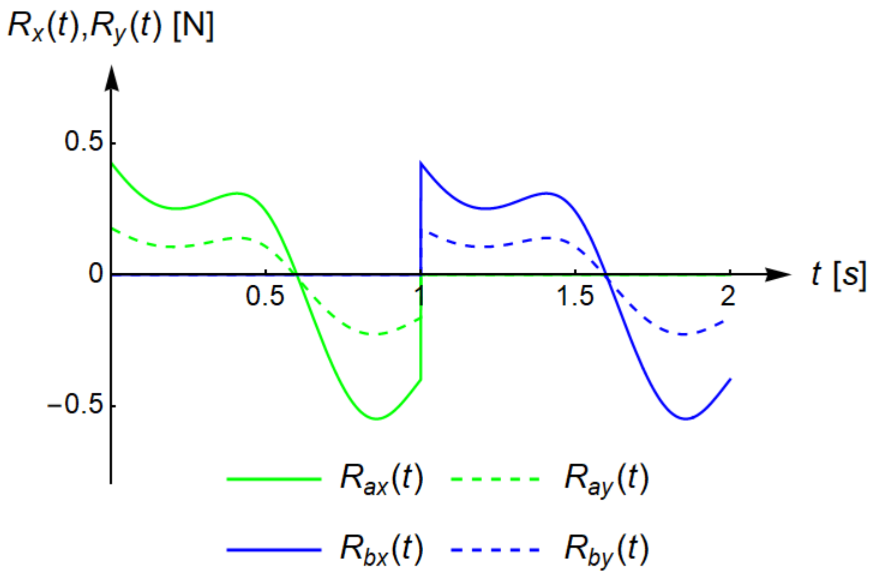
$$\mu_{ax}(t) = \frac{R_{ax}(t)}{R_{az}(t)} \qquad \mu_{ay}(t) = \frac{R_{ay}(t)}{R_{az}(t)} \quad \Rightarrow \quad \mu_a(t) = \frac{\sqrt{R_{ax}^2(t) + R_{ay}^2(t)}}{R_{az}(t)}$$

$$\mu_{bx}(t) = \frac{R_{bx}(t)}{R_{bz}(t)} \qquad \mu_{by}(t) = \frac{R_{by}(t)}{R_{bz}(t)} \quad \Rightarrow \quad \mu_b(t) = \frac{\sqrt{R_{bx}^2(t) + R_{by}^2(t)}}{R_{bz}(t)}$$

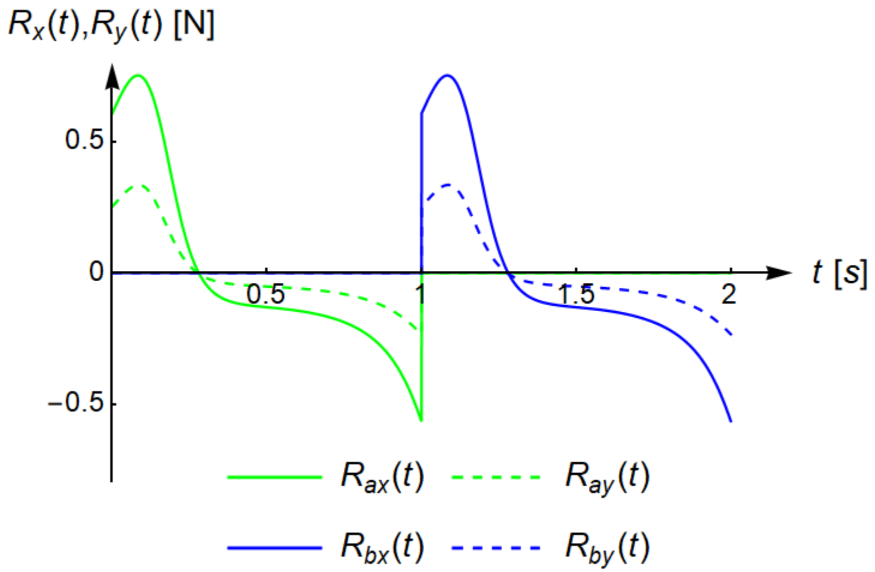
CPG1



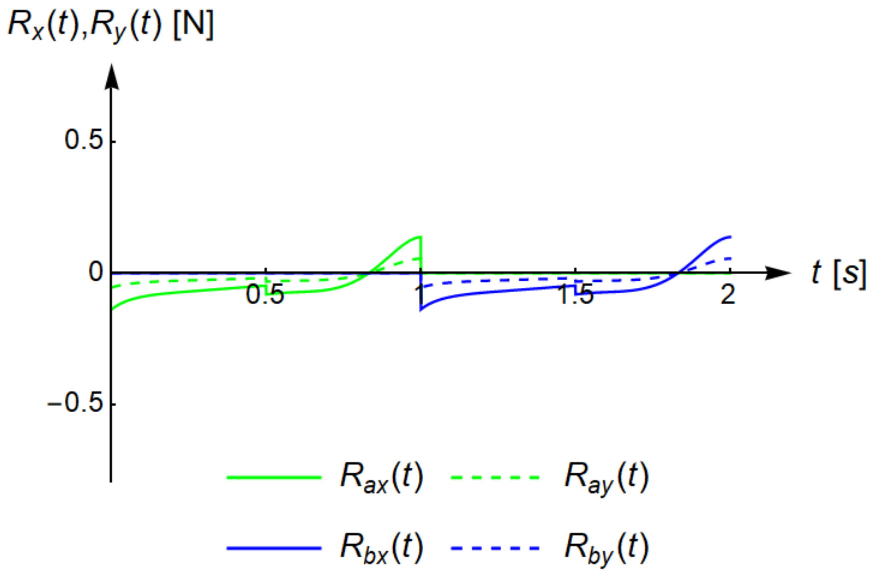
CPG2



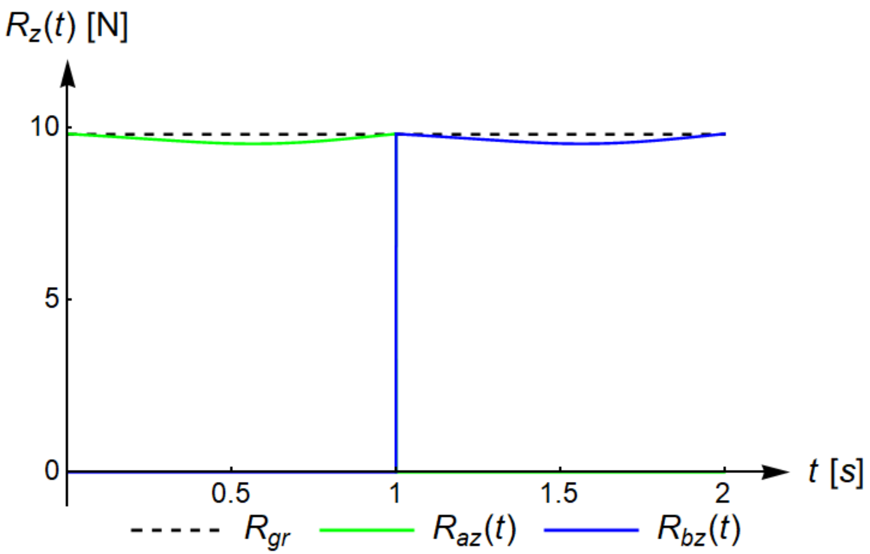
CPG3



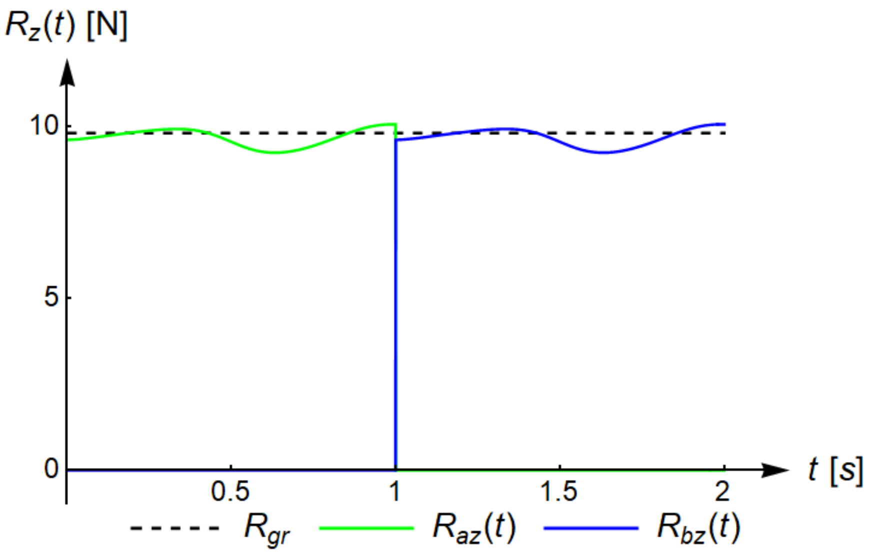
CPG4



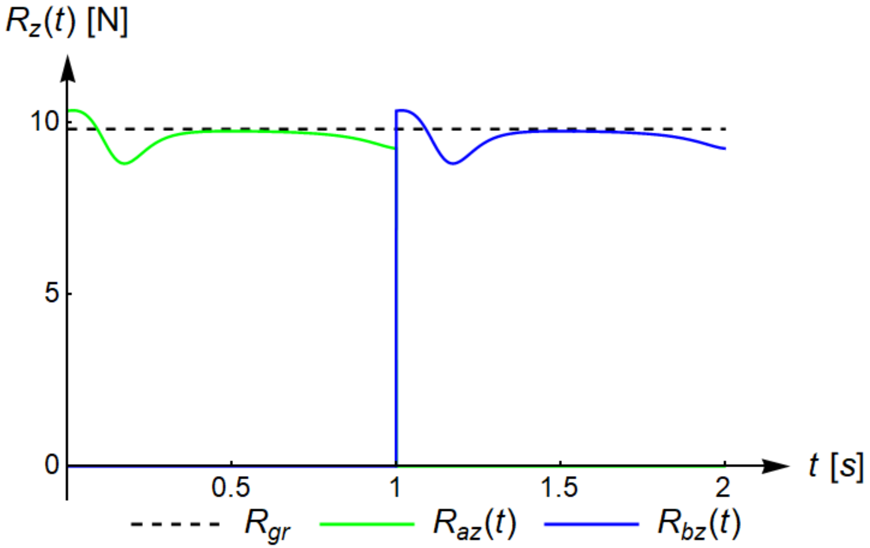
CPG1



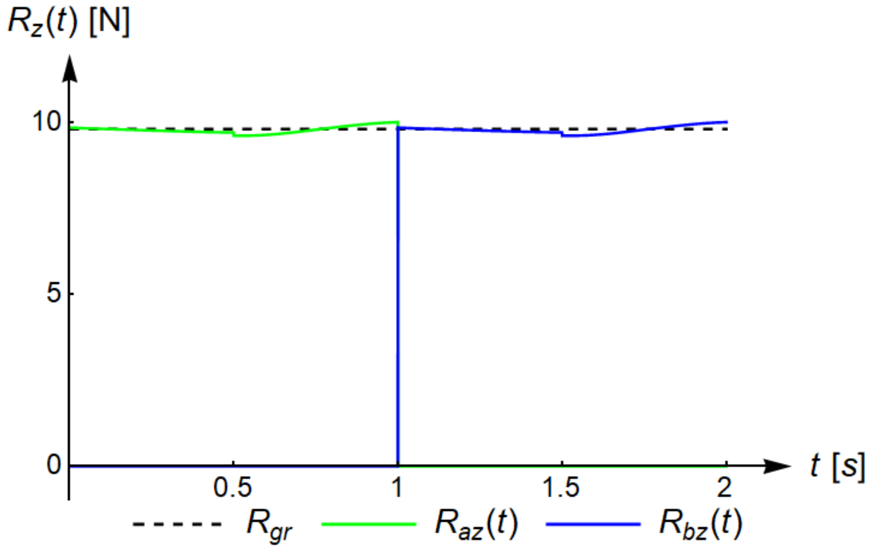
CPG2

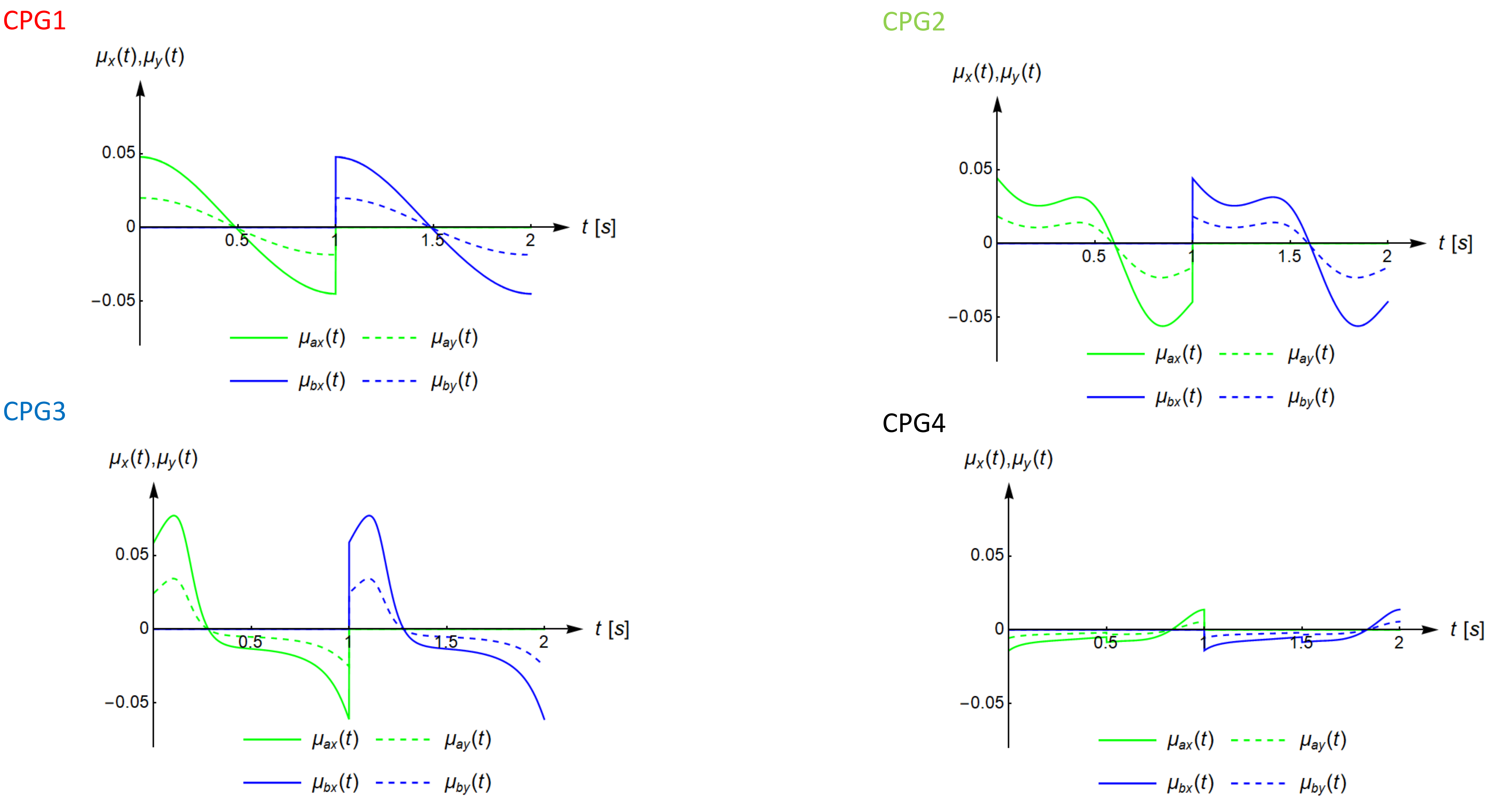


CPG3

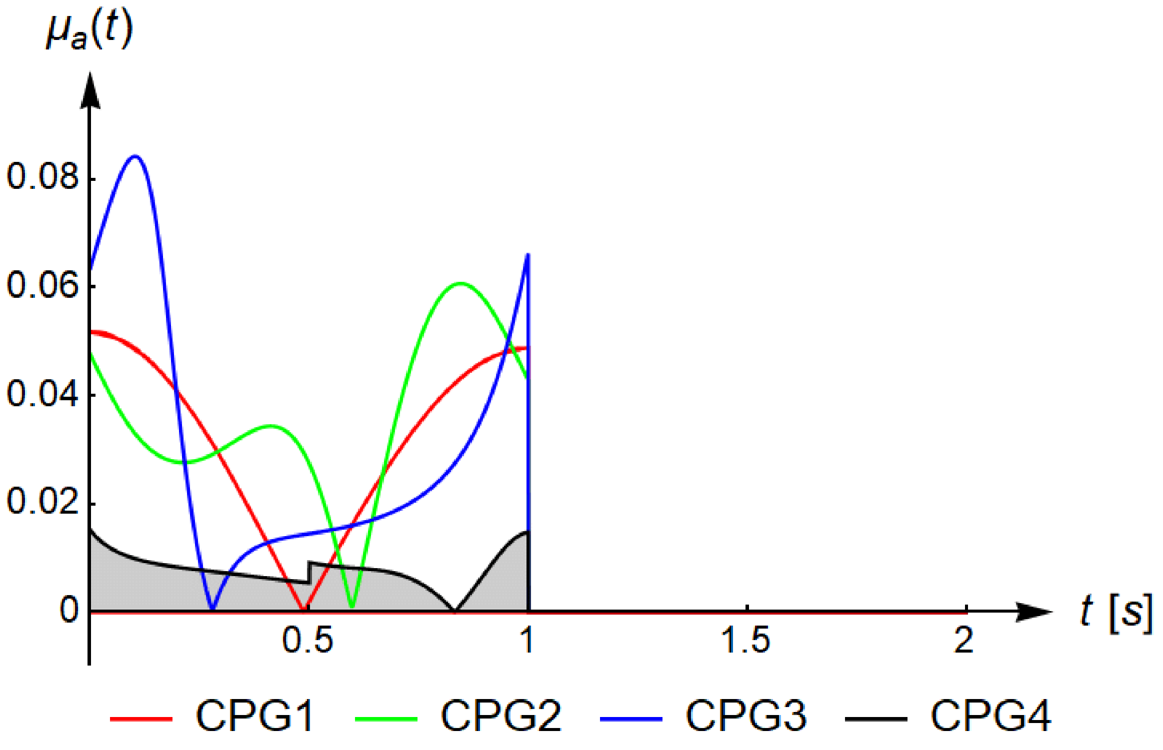


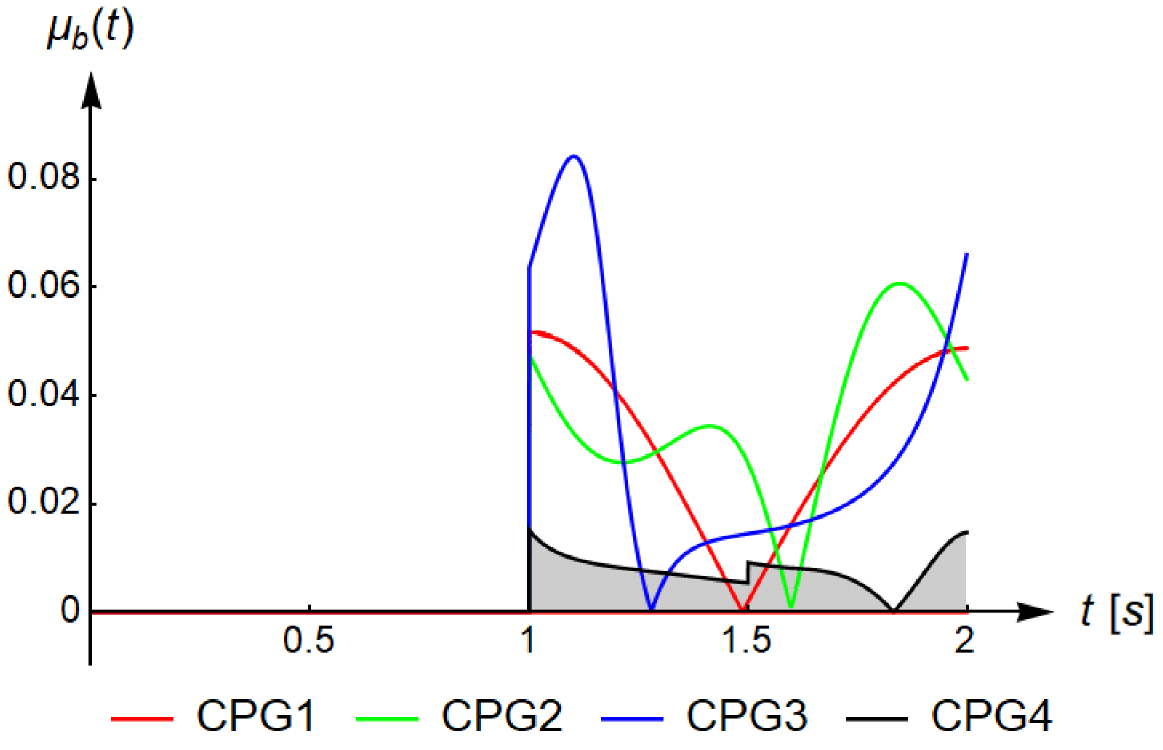
CPG4



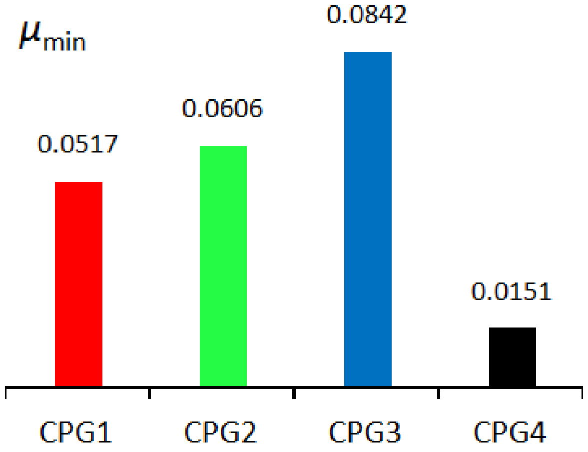


CPG1
$\mu_x(t), \mu_y(t)$
0.05
0
−0.05
0.5
1
1.5
2
t [s]
$\mu_{ax}(t)$
$\mu_{ay}(t)$
$\mu_{bx}(t)$
$\mu_{by}(t)$
CPG2
$\mu_x(t), \mu_y(t)$
0.05
0
−0.05
0.5
1
1.5
2
t [s]
$\mu_{ax}(t)$
$\mu_{ay}(t)$
$\mu_{bx}(t)$
$\mu_{by}(t)$
CPG3
$\mu_x(t), \mu_y(t)$
0.05
0
−0.05
0.5
1
1.5
2
t [s]
$\mu_{ax}(t)$
$\mu_{ay}(t)$
$\mu_{bx}(t)$
$\mu_{by}(t)$
CPG4
$\mu_x(t), \mu_y(t)$
0.05
0
−0.05
0.5
1
1.5
2
t [s]
$\mu_{ax}(t)$
$\mu_{ay}(t)$
$\mu_{bx}(t)$
$\mu_{by}(t)$



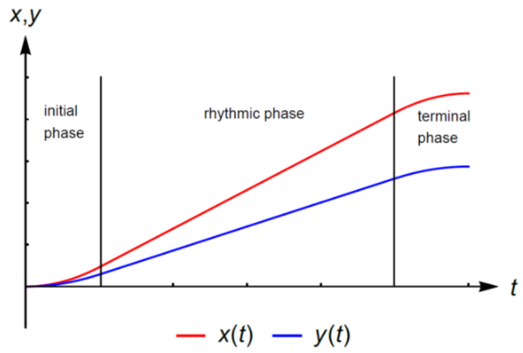


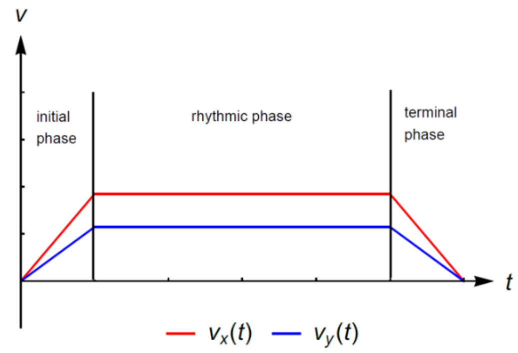
$$\mu_{\min} = \max_{t \in [0.T]} \mu_a(t) = \max_{t \in [0.T]} \mu_b(t)$$



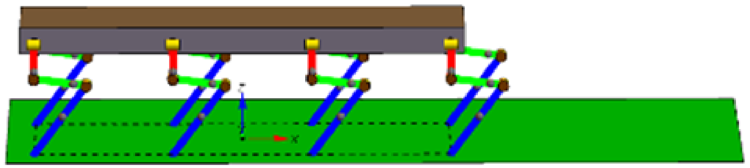
$\theta = \pi/8$

$$P(t)=\begin{cases}\dfrac{2}{T}t & \text{if} \quad t\in[0,\ 0.5T],\\ 1 & \text{if} \quad t\in(0.5T,\ (n+0.5)T],\\ 1-\dfrac{2}{T}(t-(n+0.5)T) & \text{if} \quad t\in((n+0.5)T,\ (n+1)T].\end{cases}$$

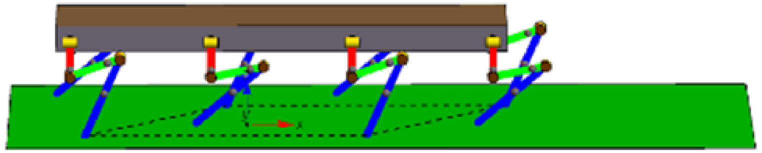




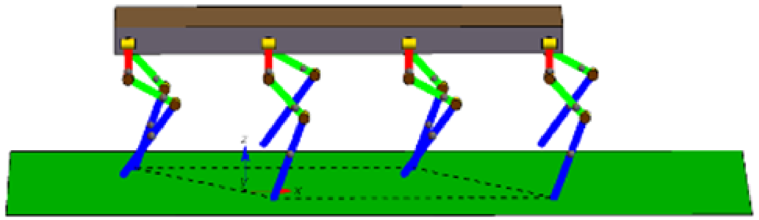
$h(t) = (0.21 - 0.1e^{-0.3t}\sin 2t + 0.015t)$ m



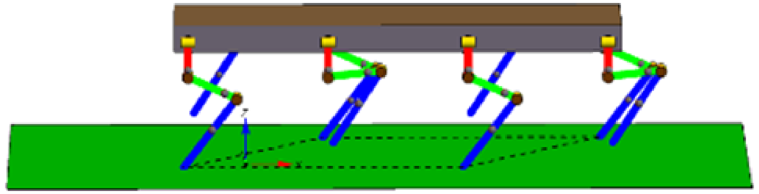
$t = 0$ s



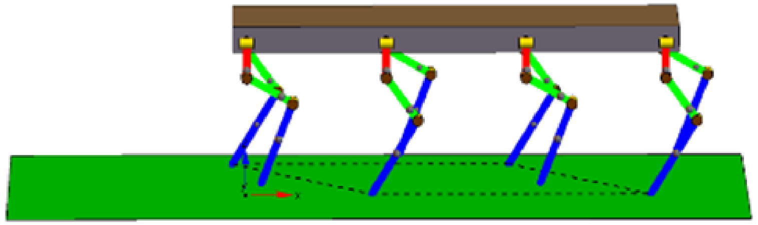
$t = 1.2$ s



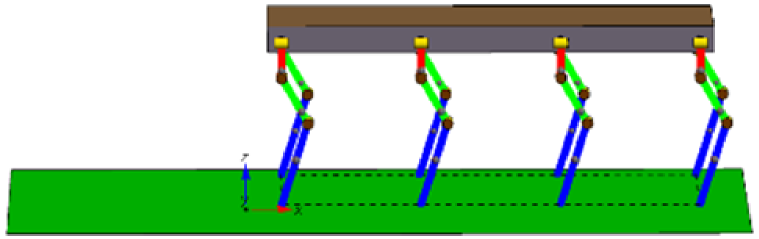
$t = 2.4$ s



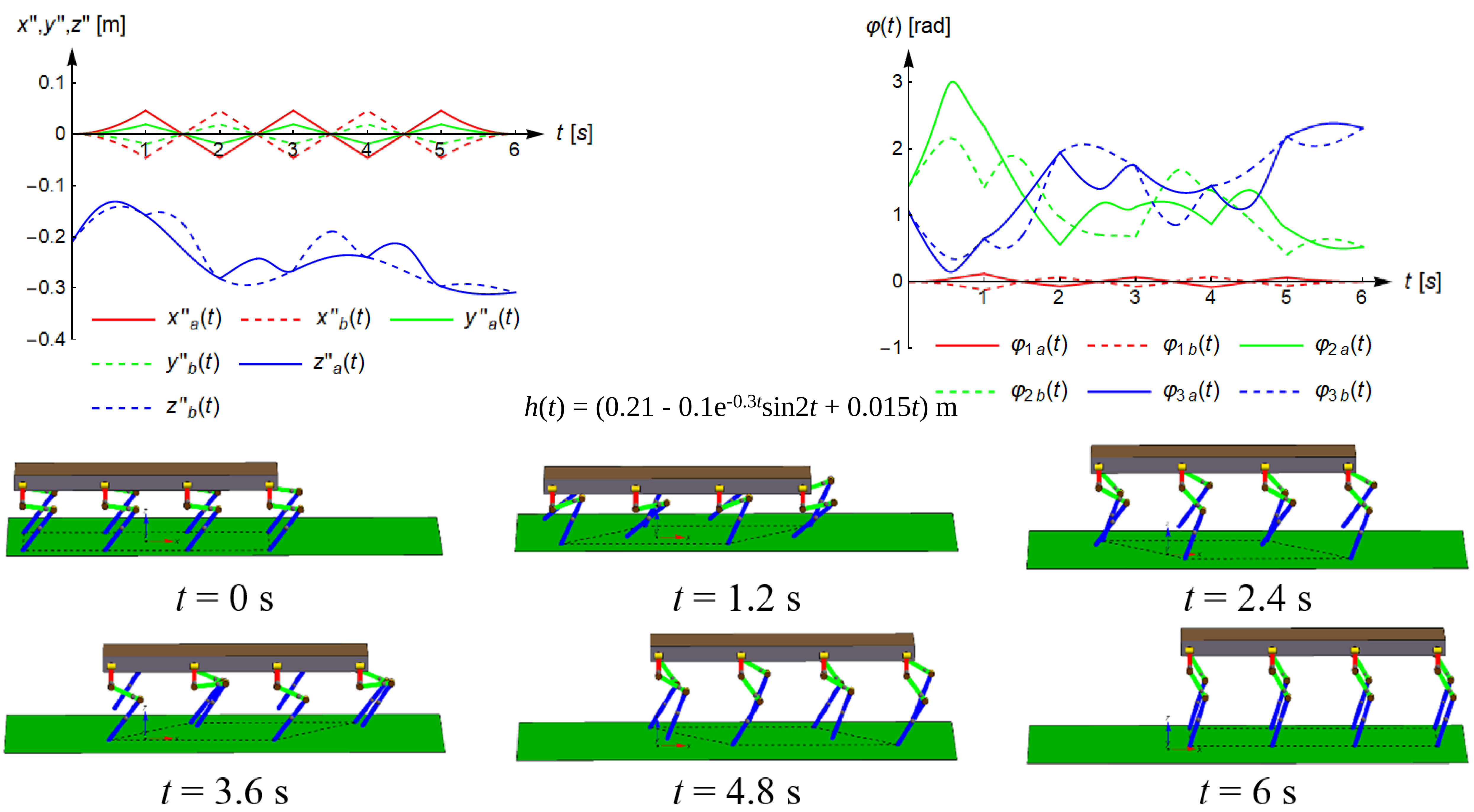
$t = 3.6$ s



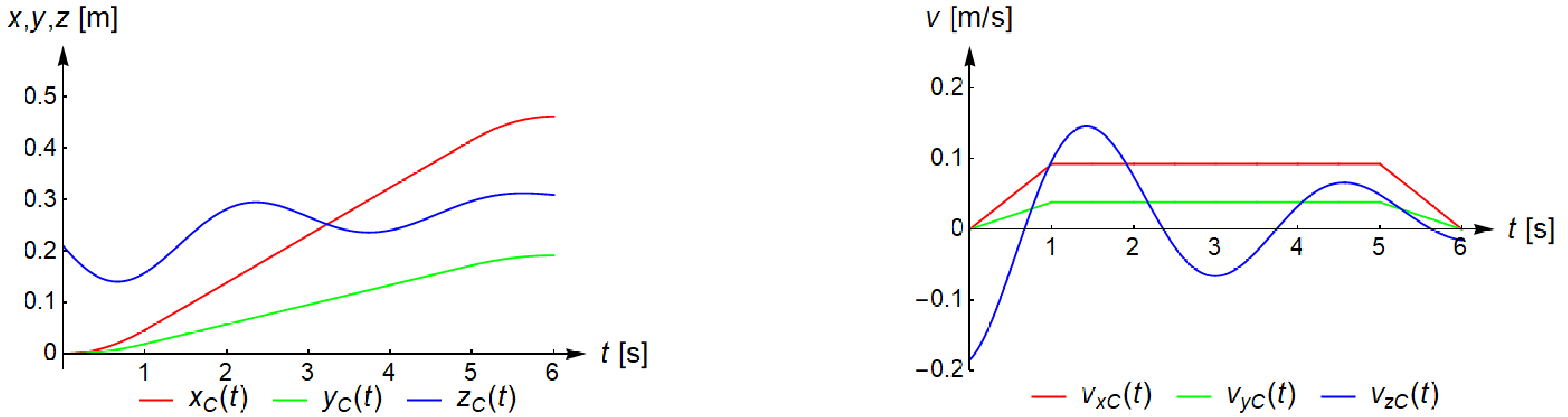
$t = 4.8$ s



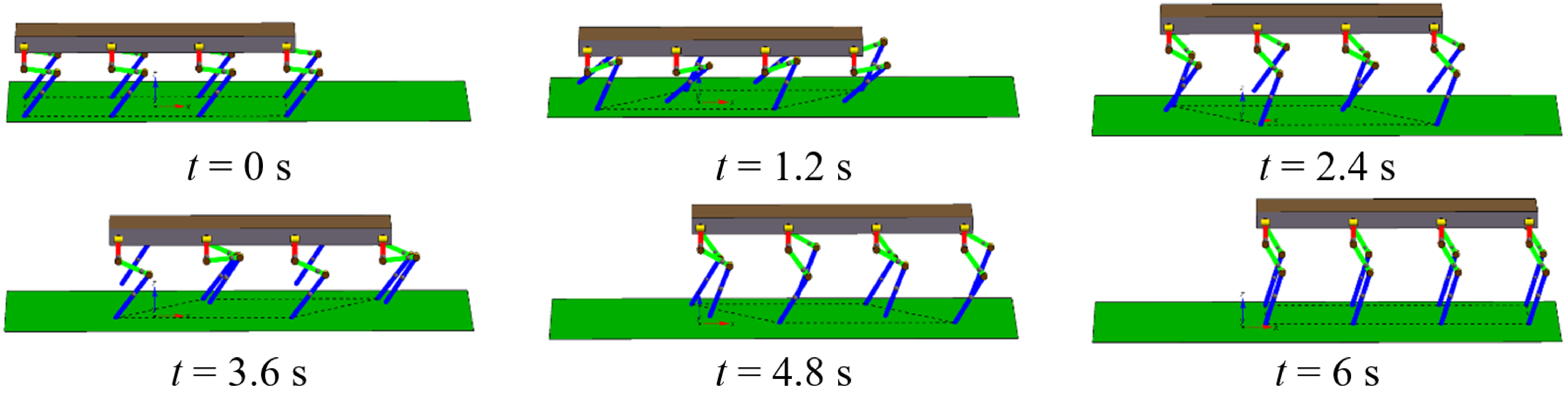
$t = 6$ s



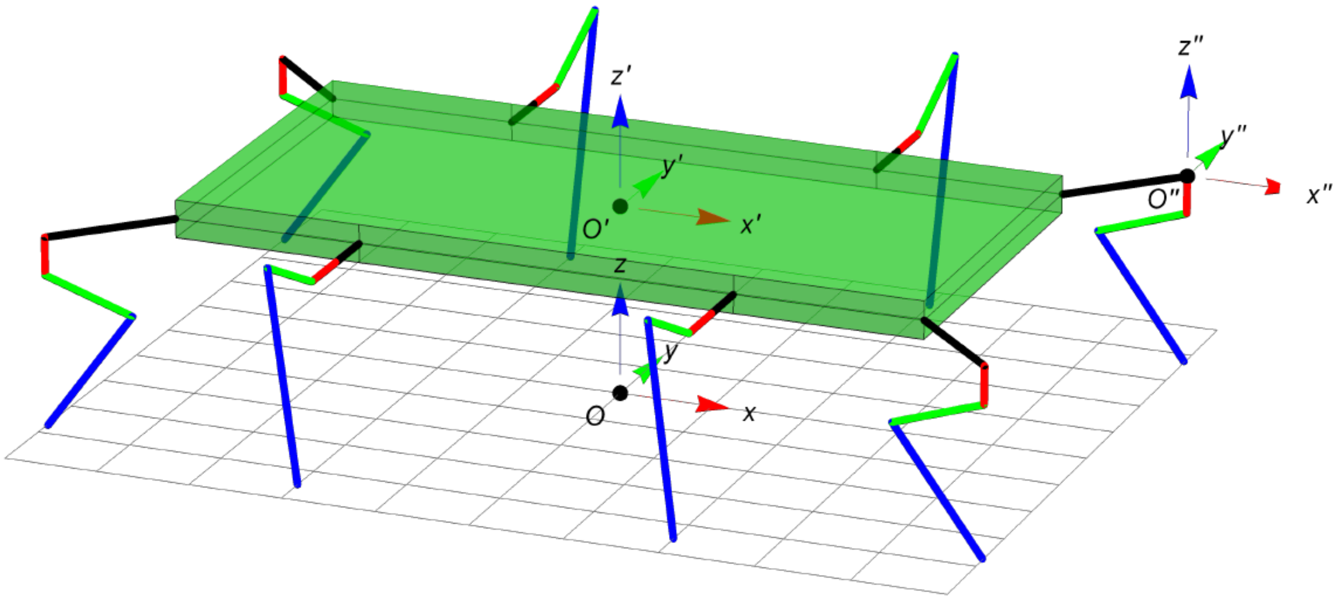
x",y",z" [m]
0.1
0
−0.1
−0.2
−0.3
−0.4
1
2
3
4
5
6
t [s]
x"a(t)
x"b(t)
y"a(t)
y"b(t)
z"a(t)
z"b(t)
φ(t) [rad]
3
2
1
0
−1
1
2
3
4
5
6
t [s]
φ1 a(t)
φ1 b(t)
φ2 a(t)
φ2 b(t)
φ3 a(t)
φ3 b(t)
h(t) = (0.21 - 0.1e-0.3tsin2t + 0.015t) m
t = 0 s
t = 1.2 s
t = 2.4 s
t = 3.6 s
t = 4.8 s
t = 6 s



$h(t) = (0.21 - 0.1e^{-0.3t}\sin 2t + 0.015t)$ m



General model of the robot:



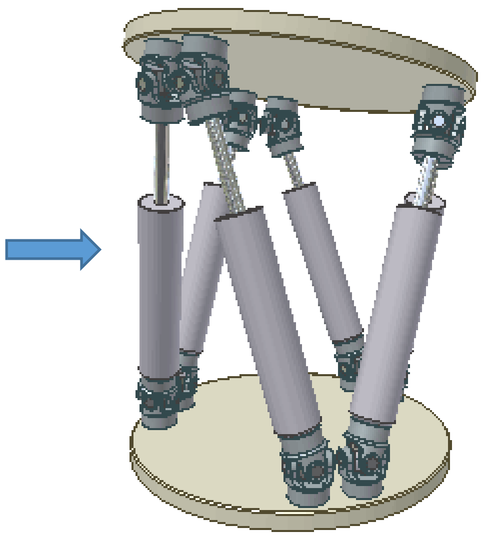
$$z = z_G(x, y, t)$$

$$\Delta\mathbf{r}_G(t) = \left[\Delta x_G(t), \Delta y_G(t), \Delta z_G(t)\right]^{\mathrm{T}}$$

$$\mathbf{R}_G^{(x)}(\alpha_G(t)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_G(t) & -\sin\alpha_G(t) \\ 0 & \sin\alpha_G(t) & \cos\alpha_G(t) \end{bmatrix}$$

$$\mathbf{R}_G^{(y)}(\beta_G(t)) = \begin{bmatrix} \cos\beta_G(t) & 0 & \sin\beta_G(t) \\ 0 & 1 & 0 \\ -\sin\beta_G(t) & 0 & \cos\beta_G(t) \end{bmatrix}$$

$$\mathbf{R}_G^{(z)}(\gamma_G(t)) = \begin{bmatrix} \cos\gamma_G(t) & -\sin\gamma_G(t) & 0 \\ \sin\gamma_G(t) & \cos\gamma_G(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



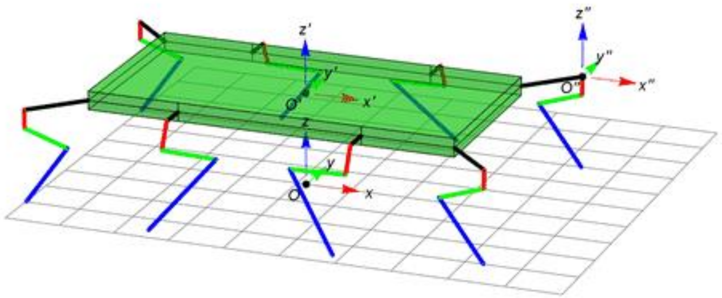
$$\mathbf{R}_R^{(x)}(\alpha_R(t)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_R(t) & -\sin\alpha_R(t) \\ 0 & \sin\alpha_R(t) & \cos\alpha_R(t) \end{bmatrix}$$

$$\mathbf{R}_R^{(y)}(\beta_R(t)) = \begin{bmatrix} \cos\beta_R(t) & 0 & \sin\beta_R(t) \\ 0 & 1 & 0 \\ -\sin\beta_R(t) & 0 & \cos\beta_R(t) \end{bmatrix}$$

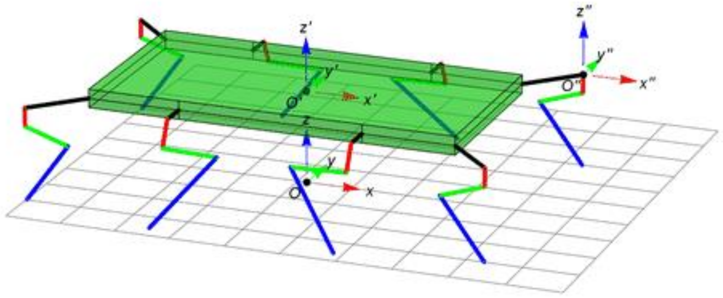
$$\mathbf{R}_R^{(z)}(\gamma_R(t)) = \begin{bmatrix} \cos\gamma_R(t) & -\sin\gamma_R(t) & 0 \\ \sin\gamma_R(t) & \cos\gamma_R(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta\mathbf{r}_R(t) = \left[\Delta x_R(t), \Delta y_R(t), \Delta z_R(t)\right]^{\mathrm{T}}$$

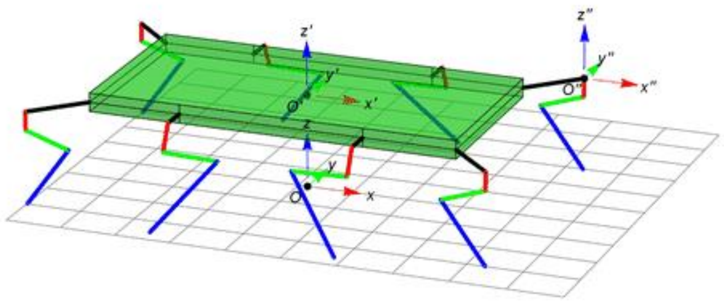
Control during standing



$\Delta\alpha_R(t) \neq 0$

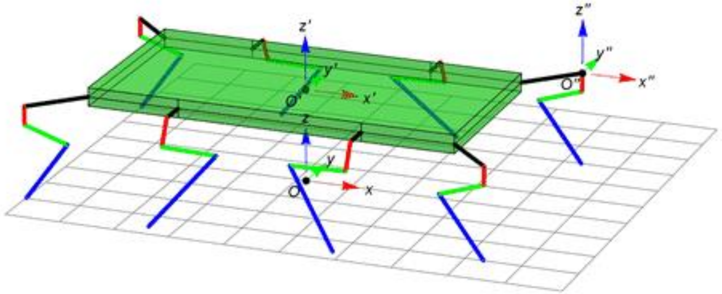


$\Delta\beta_R(t) \neq 0$

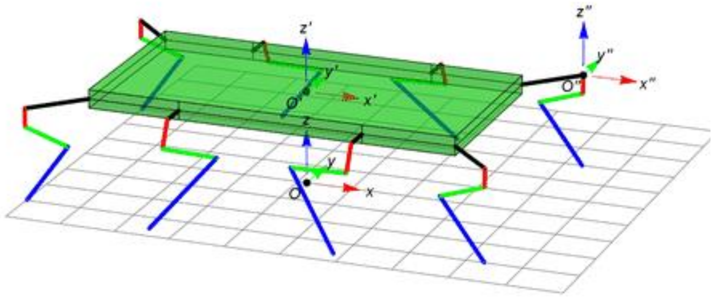


$\Delta\gamma_R(t) \neq 0$

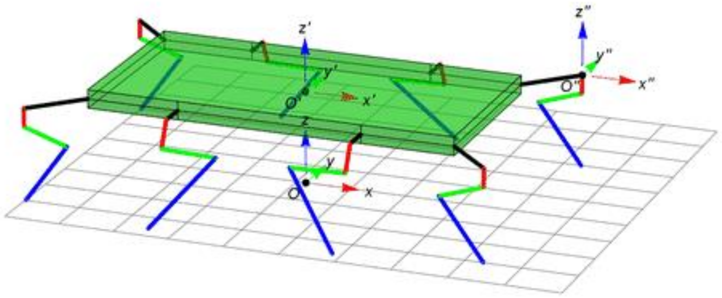
Control during walking



$\Delta\alpha_R(t) \neq 0$

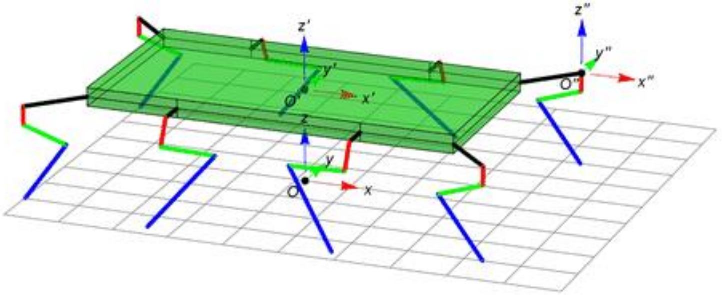


$\Delta\beta_R(t) \neq 0$

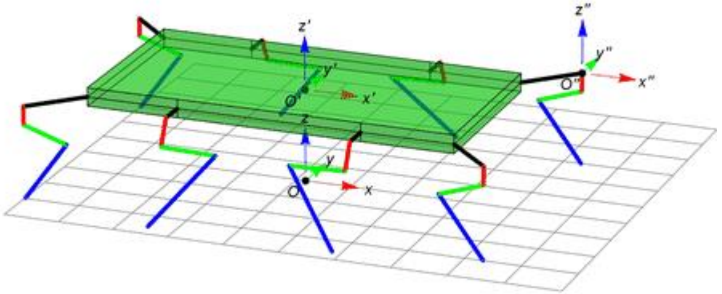


$\Delta\gamma_R(t) \neq 0$

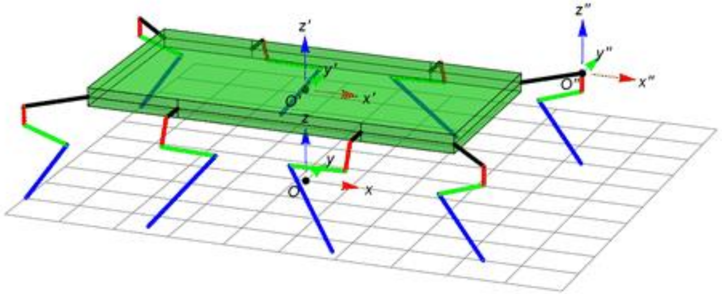
Control during standing



$\Delta x(t) \neq 0$

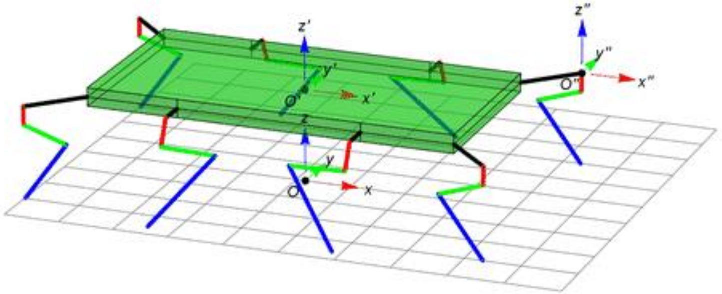


$\Delta y(t) \neq 0$

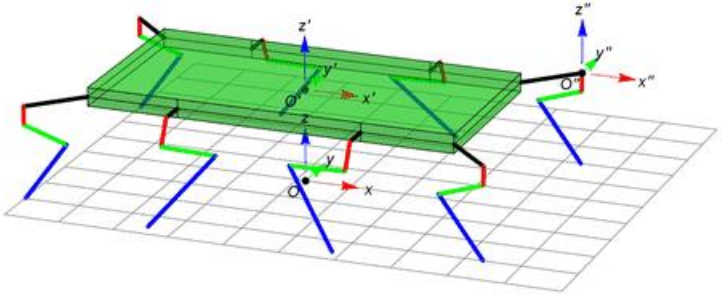


$\Delta z(t) \neq 0$

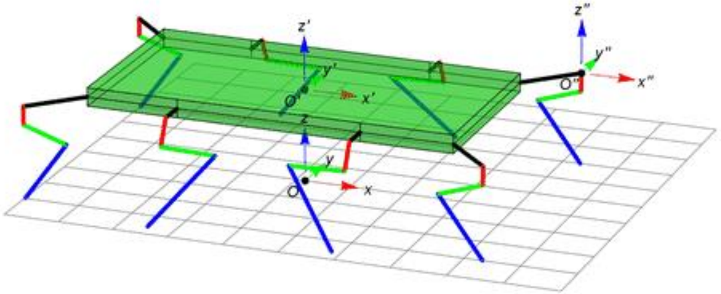
Control during walking



$\Delta x(t) \neq 0$

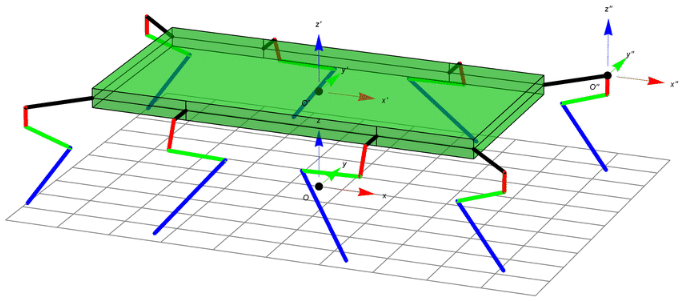


$\Delta y(t) \neq 0$

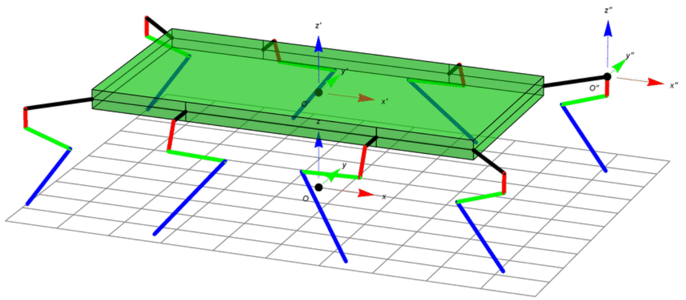


$\Delta z(t) \neq 0$

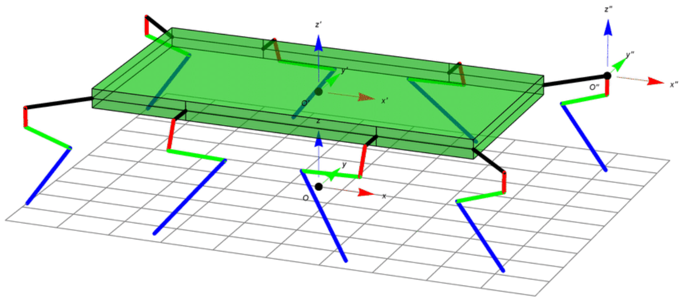
Control on vibrating ground



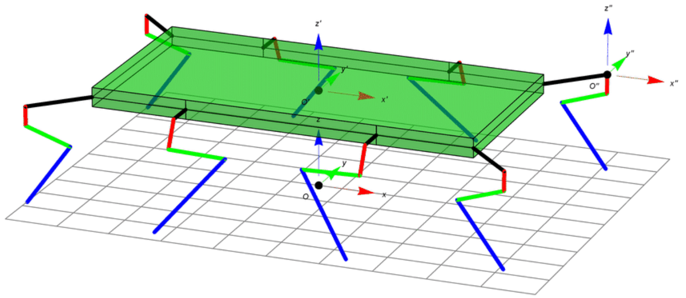
$\Delta\alpha_R(t) \neq 0$



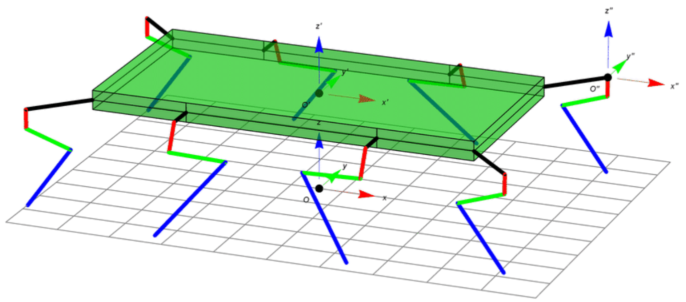
$\Delta\beta_R(t) \neq 0$



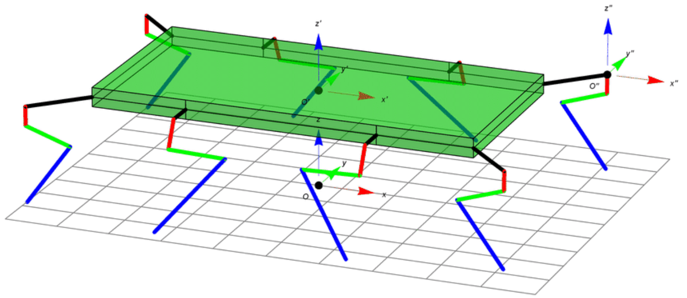
$\Delta\gamma_R(t) \neq 0$



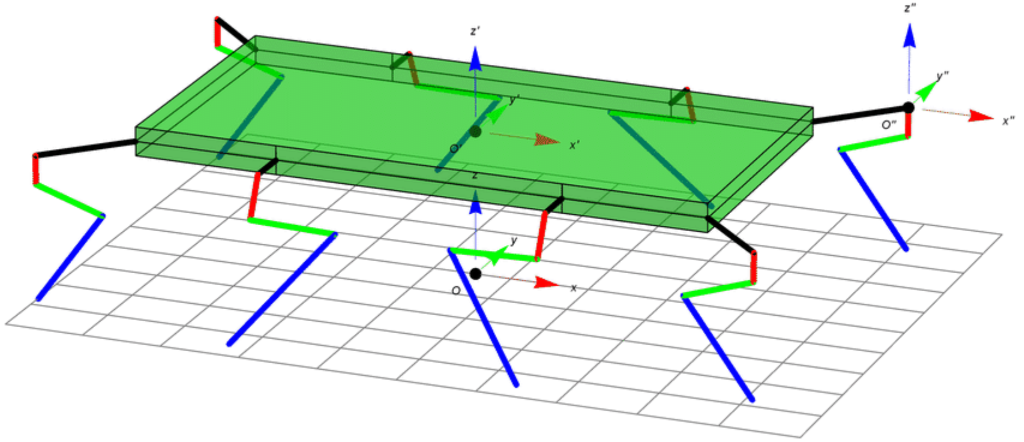
$\Delta x(t) \neq 0$

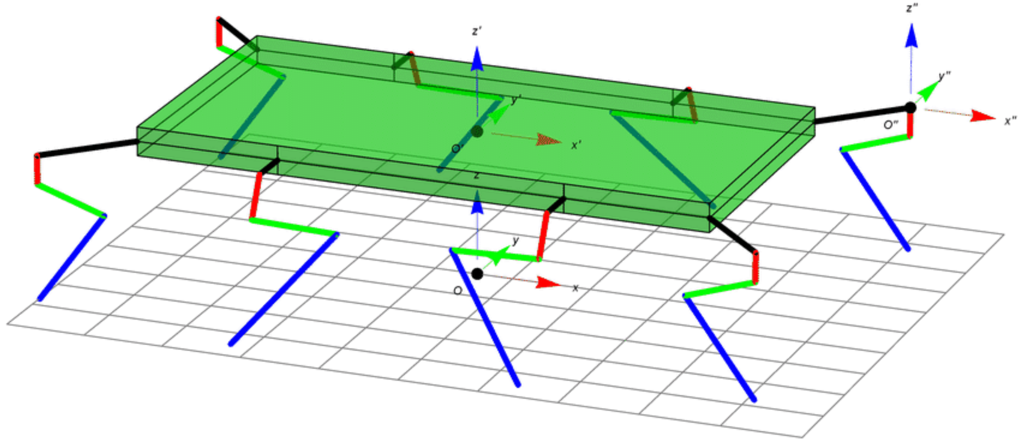


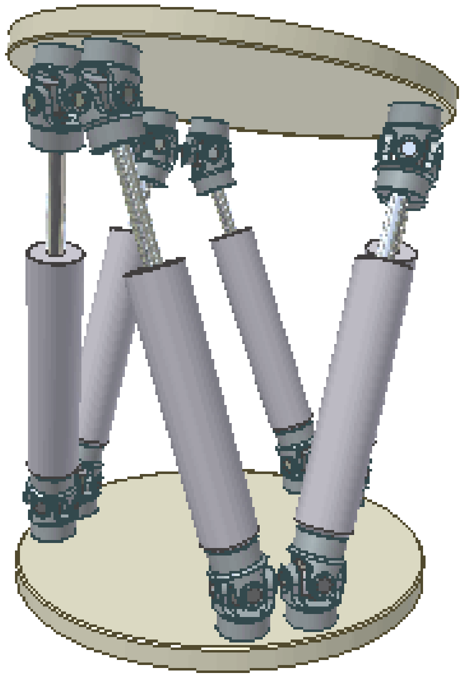
$\Delta y(t) \neq 0$

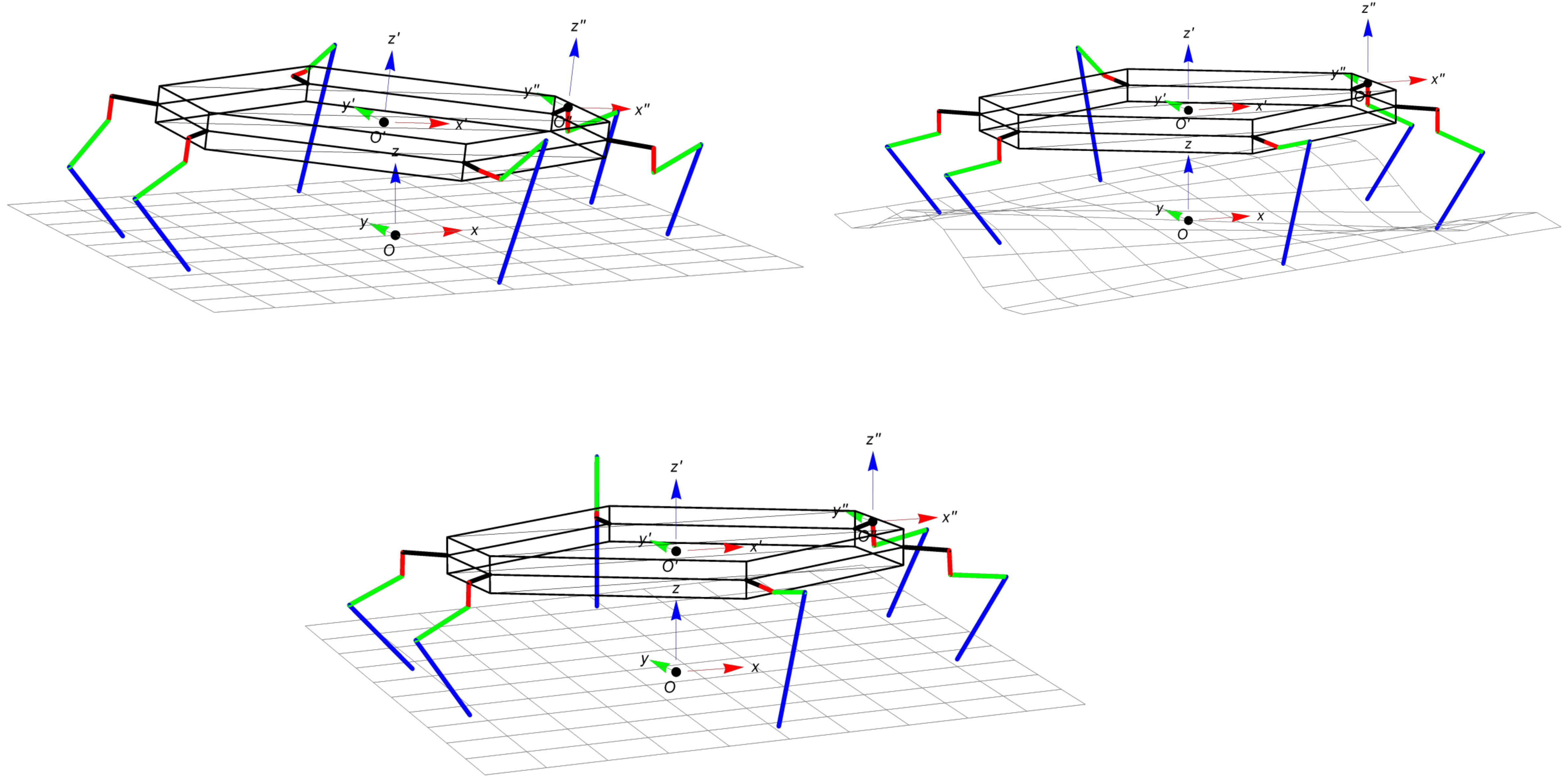


$\Delta z(t) \neq 0$

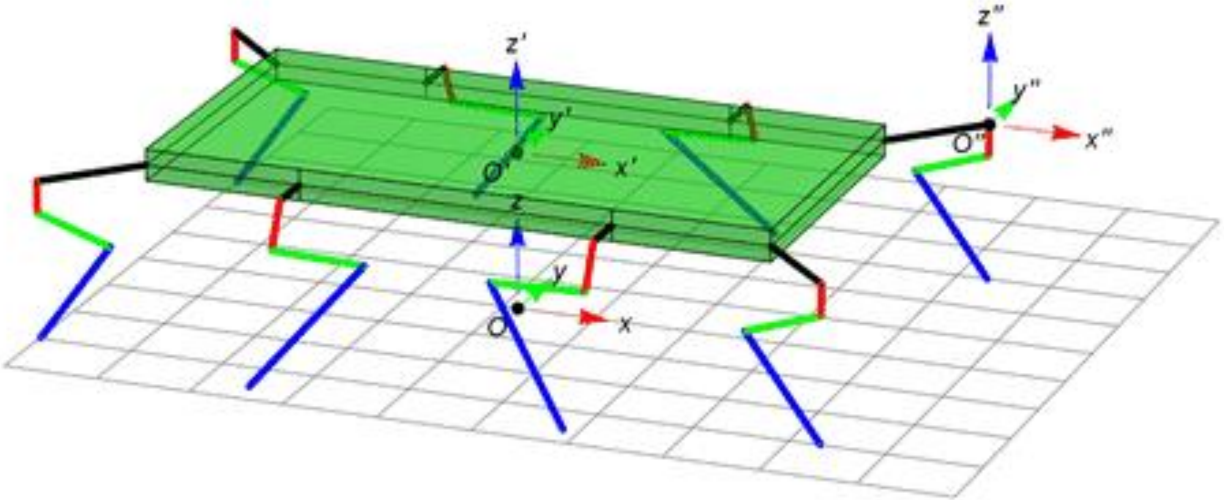


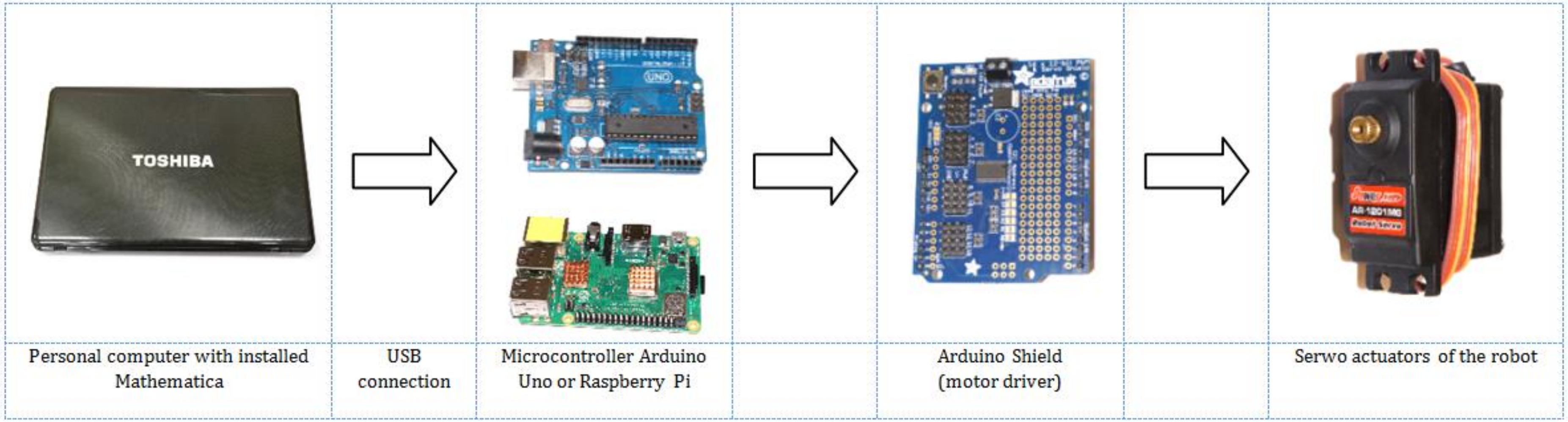






z'
z''
y'
x'
O'
y''
x''
O''
z
y
x
O


z'
y'
x'
O'
z
y
x
O
z''
y''
x''
O''


TOSHIBA
Personal computer with installed Mathematica
USB connection
Microcontroller Arduino Uno or Raspberry Pi
Arduino Shield (motor driver)
Serwo actuators of the robot