



Politechnika Łódzka

Wydział Mechaniczny



PRELEKCJA NAUKOWA

Modelowanie matematyczne anizotropowego tarcia suchego i wahadła magnetycznego - zagadnienia wybrane z przykładami efektywnej prezentacji wyników

Wybrane modele matematyczne oddziaływania magnetycznego w układzie wahadła oraz hodografu i reguły poślizgu anizotropowego tarcia suchego. Opracowanie wykresów, grafik i animacji na potrzeby artykułów i prezentacji z wykorzystaniem środowiska Mathematica.

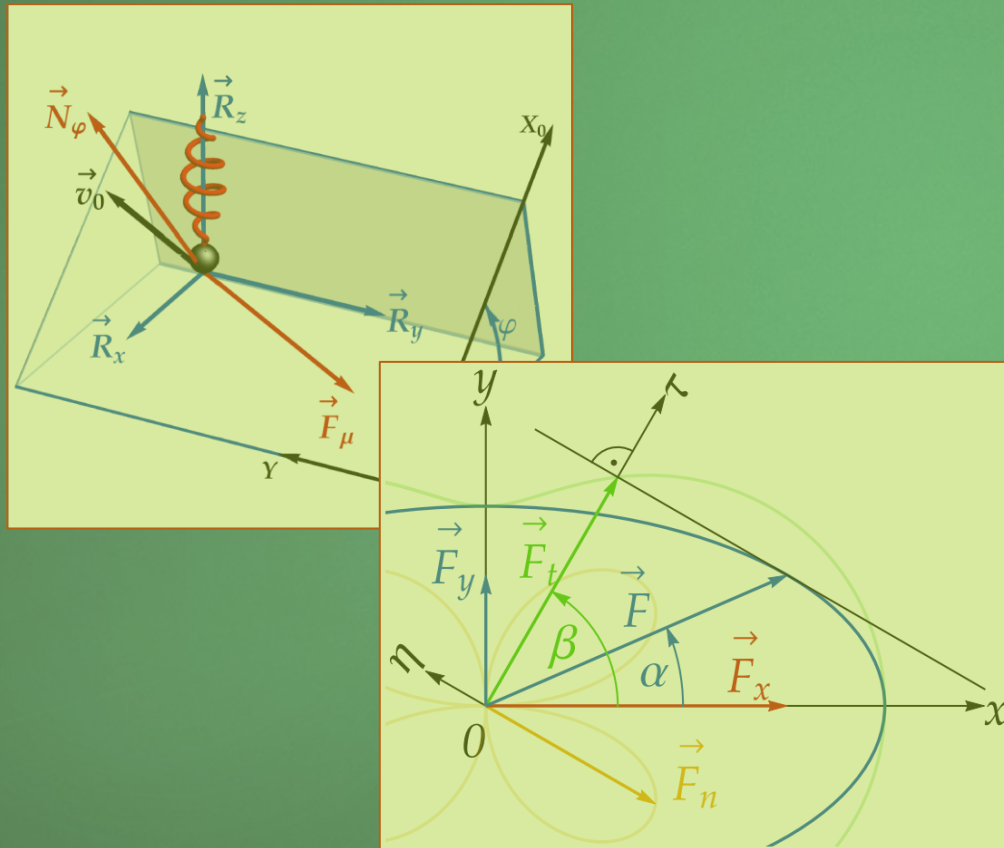


dr inż. Adam Wijata
Katedra Automatyki,
Biomechaniki i Mechatroniki

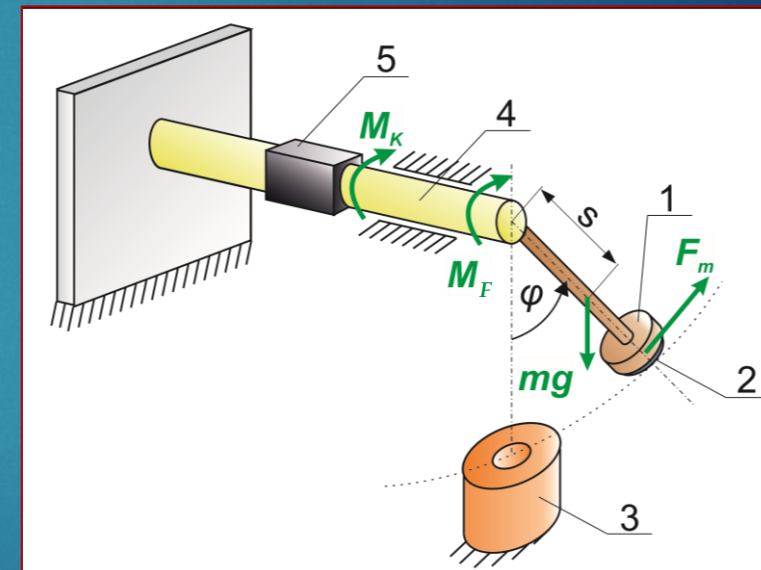
Plan prezentacji



Anizotropowe tarcie suche

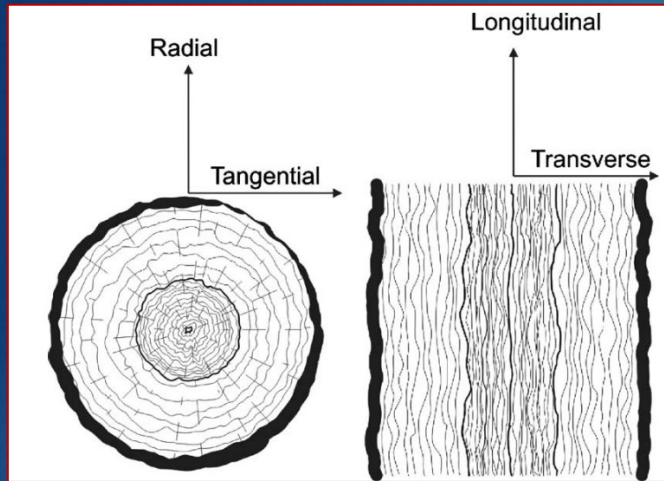


Wahadło magnetyczne

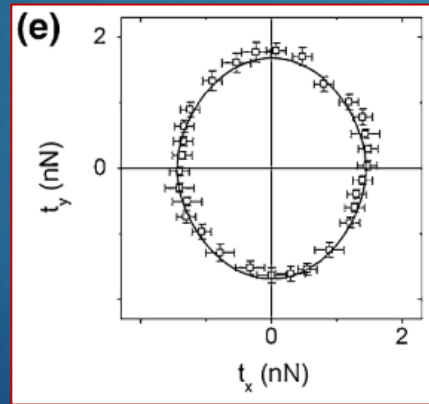
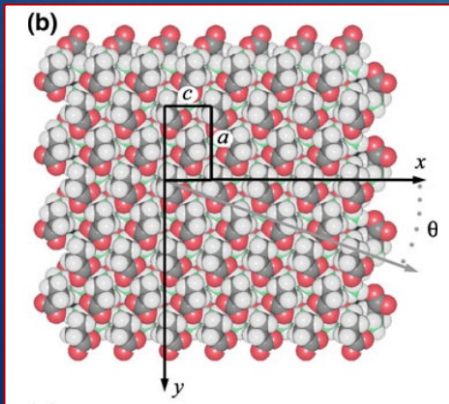


Anizotropowe tarcie suche

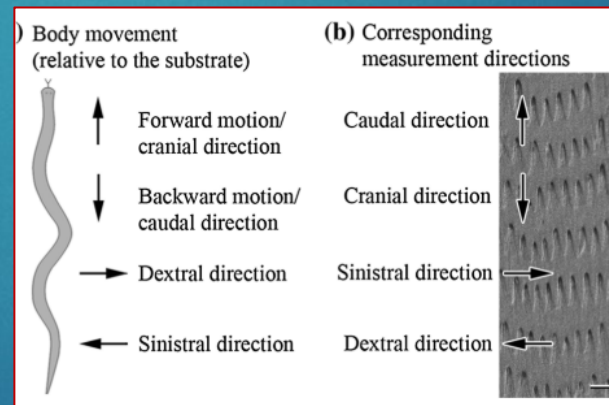
Przykłady



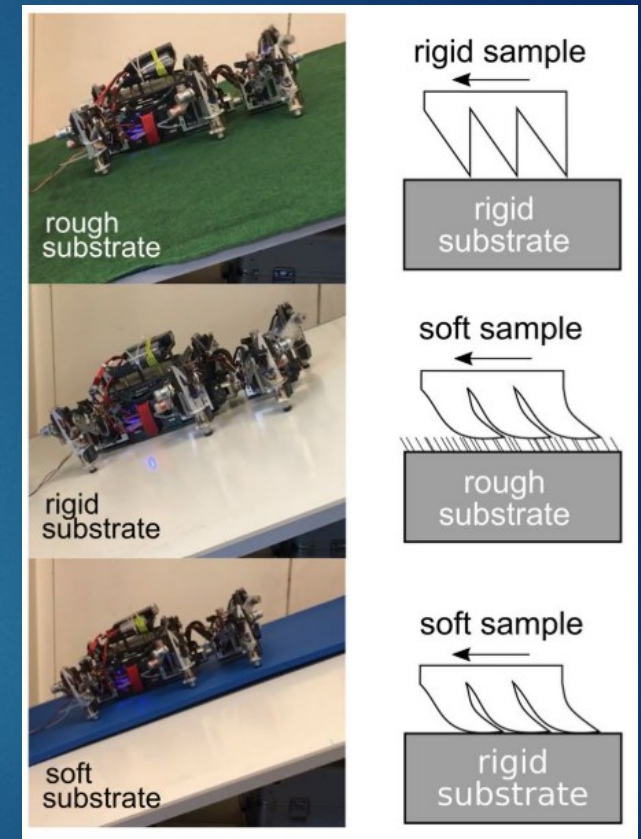
[1] A. Zmitrowicz. Models of kinematics dependent anisotropic and heterogeneous friction. *International Journal of Solids and Structures*, 43(14-15), 4407–4451 (2006).



[2] M. Campione, S. Trabattoni, M. Moret. Nanoscale Mapping of Frictional Anisotropy. *Tribology Letters*, 45(1), 219–224 (2012).



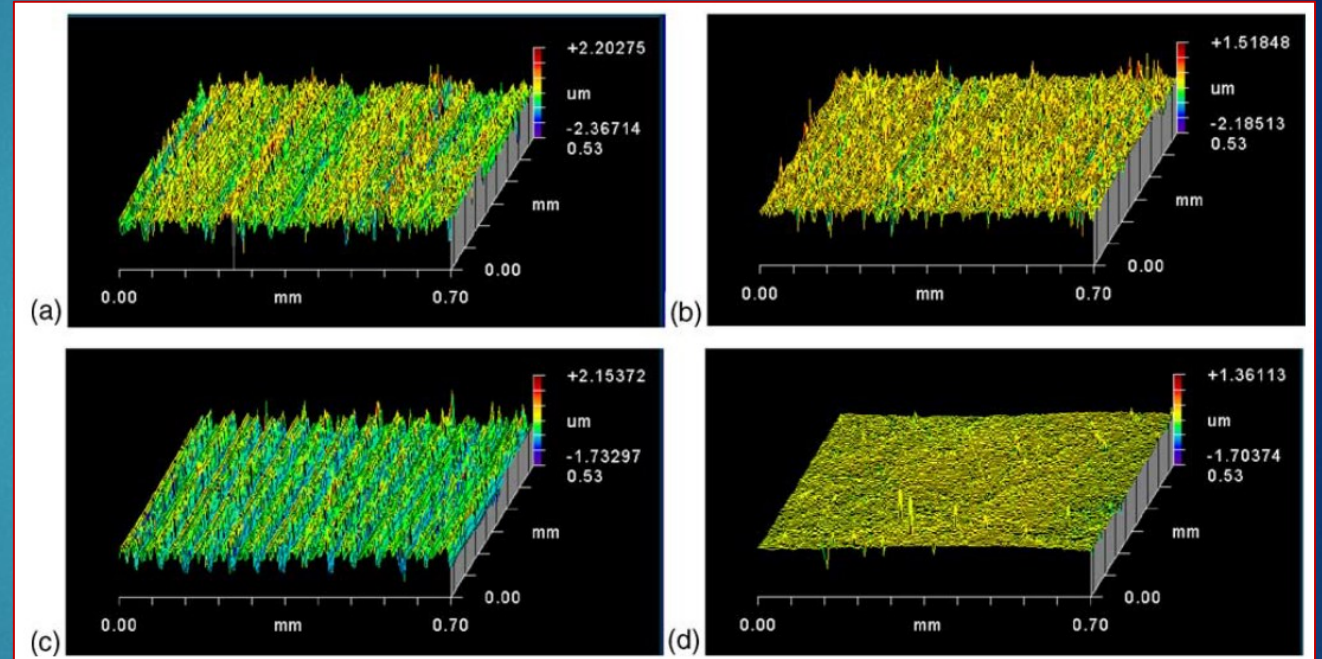
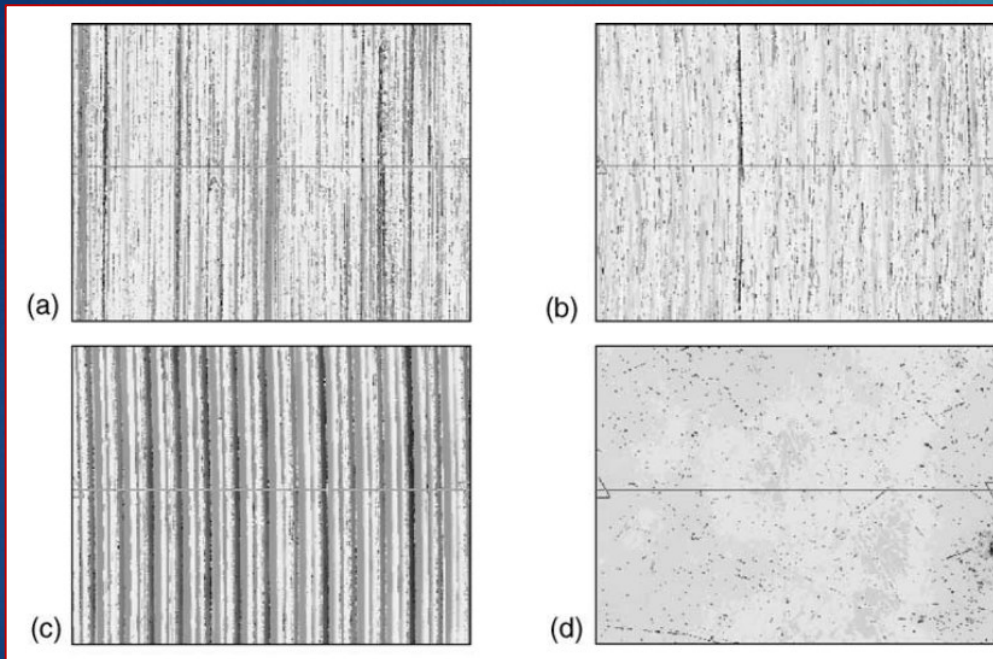
[3] M. J. Baum, *et al.*, Anisotropic Friction of the Ventral Scales in the Snake *Lampropeltis getula californica*. *Tribology Letters*, 54(2), 139–150 (2014).



[4] H. T. Tramsen, *et al.*, Getting grip in changing environments: the effect of friction anisotropy inversion on robot locomotion, *Applied Physics A*, 127(5), 1–9, (2021).

Anizotropowe tarcie suche

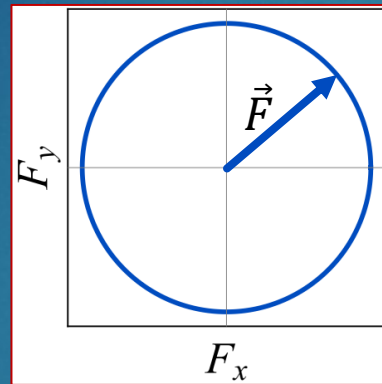
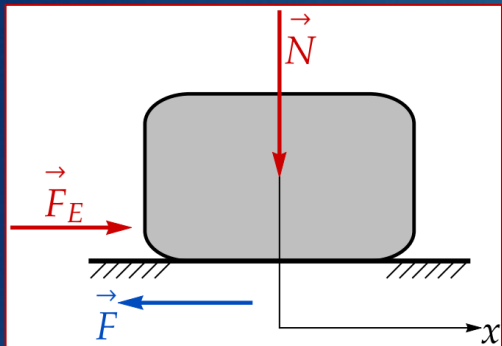
Przykłady – mikrogeometria powierzchni



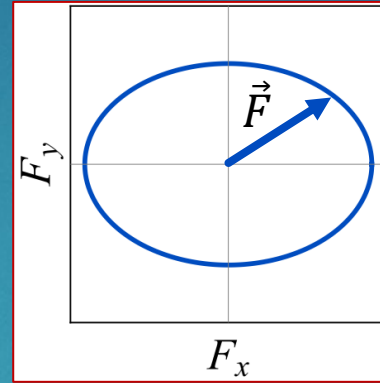
Zdjęcia oraz zmierzona topografia powierzchni po różnym rodzaju obróbki: a) szlifowana (ang. *ground*), b) honowana (ang. *honed*), c) toczona (ang. *hard turned*) i d) polerowana (ang. *isotropic finished*).

Anizotropowe tarcie suche

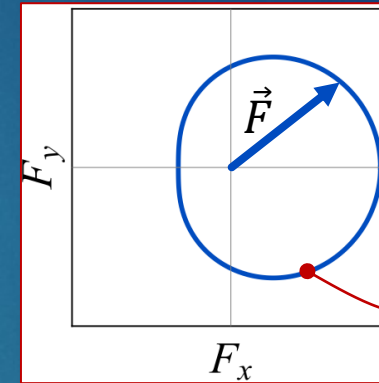
Charakterystyka tarcia dwuwymiarowego. Anizotropia.



Izotropia



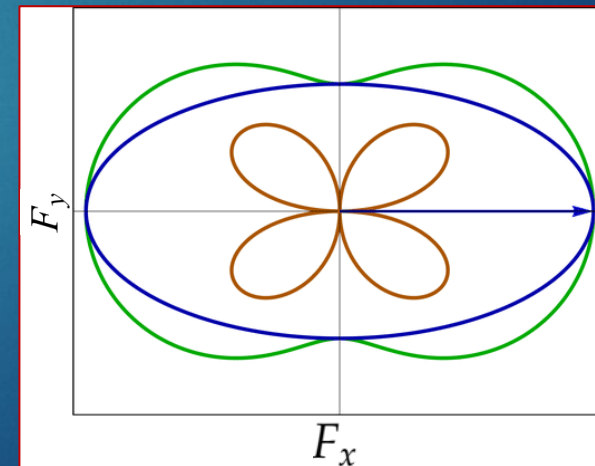
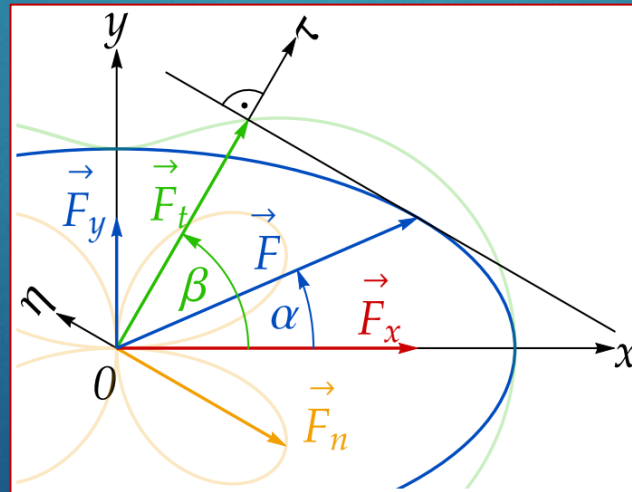
Ortotropia



Hodograf siły tarcia

Anizotropia
zwrotna

- β – kierunek poślizgu
- α – kierunek siły tarcia
- F_t – składowa styczna
- F_n – składowa normalna



Anizotropowe tarcie suche

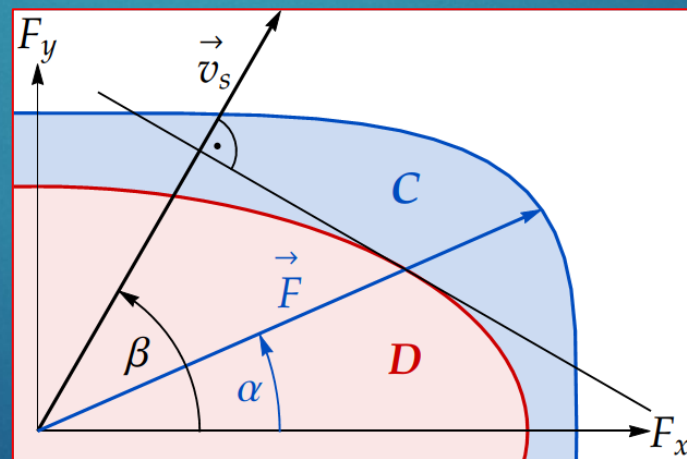
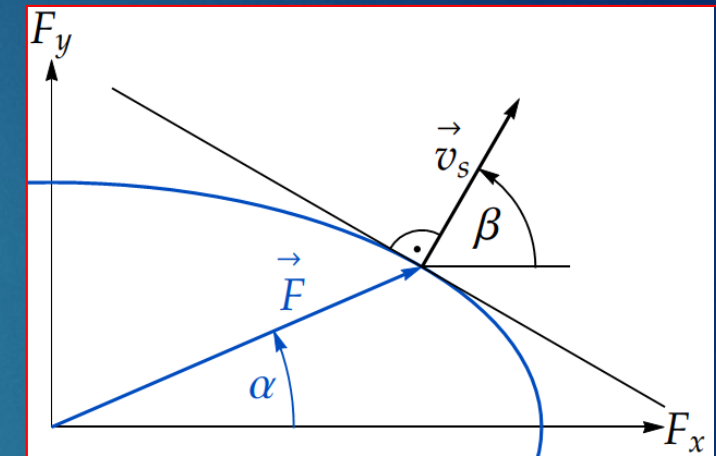
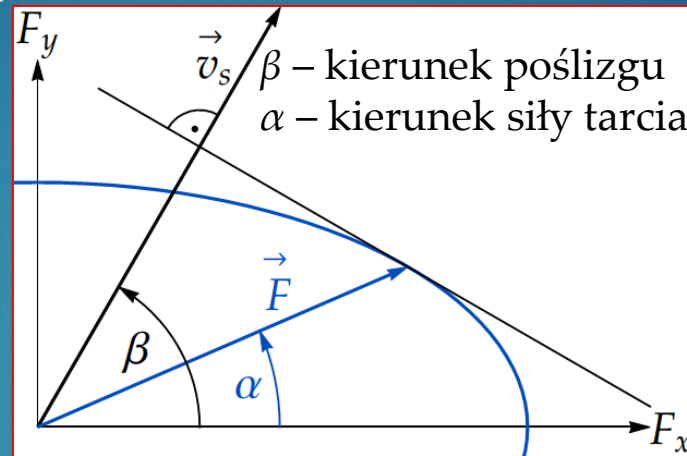
Modelowanie tarcia anizotropowego

→ Stowarzyszona reguła
Spełniony warunek największej
mocy rozpraszania energii

$$\vec{F} \cdot \vec{v}_s = \max$$

$\alpha = f(\beta)$ – reguła poślizgu

→ Niestowarzyszona reguła
poślizgu ($C \neq D$)



Hodograf C

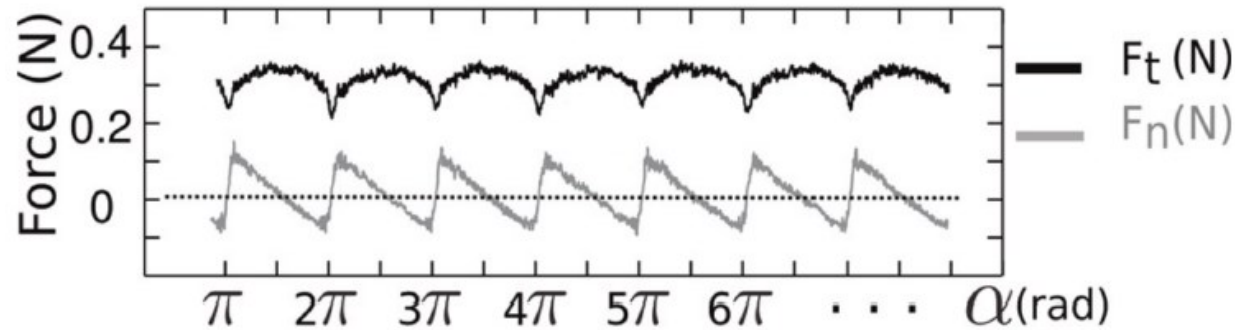
Potencjał poślizgu D

Anizotropowe tarcie suche

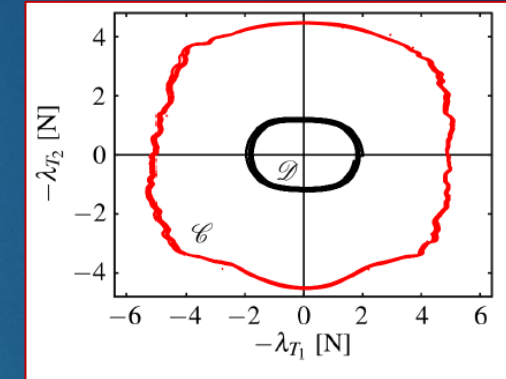
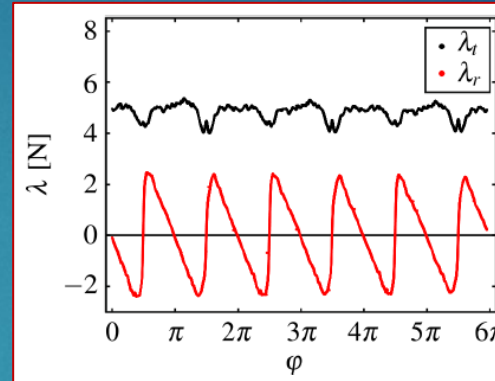
Badania doświadczalne



Filc-Tektura falista

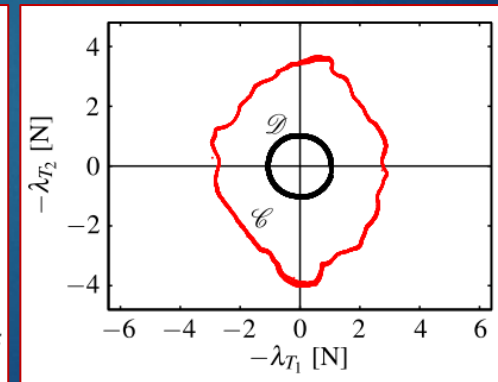
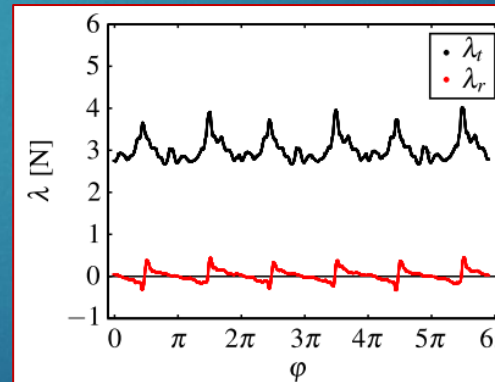
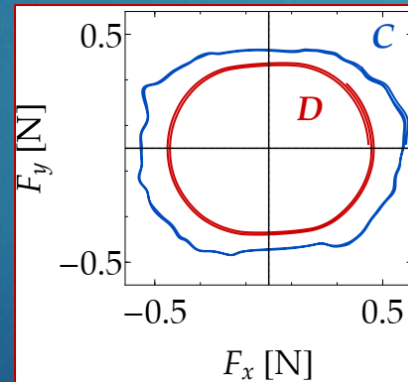
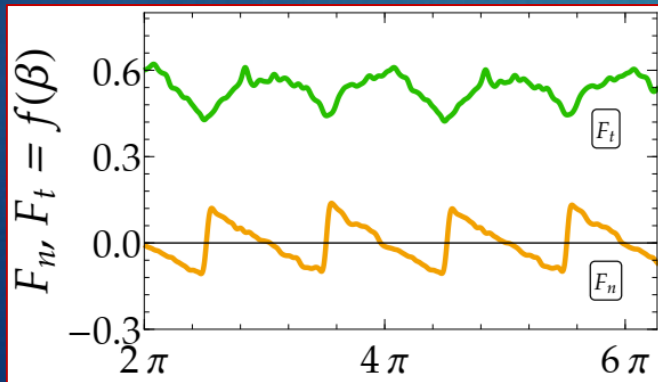


PVC - Stal



[7] F. Tapia, D. L. Tourneau, J.-C. Géminard. Anisotropic friction: assessment of force components and resulting trajectories. *EPJ Techniques and Instrumentation*, 3(1), 1 (2016).

Stal - Stal



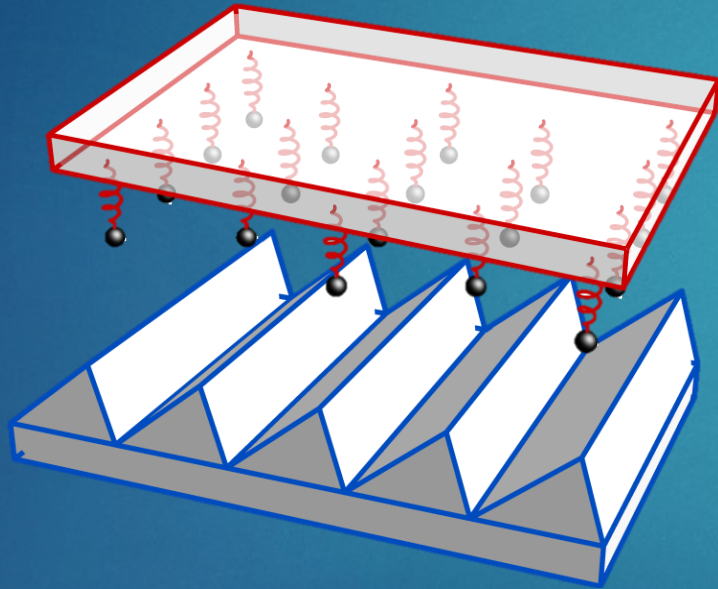
[8] Adam Wijata, Modelowanie matematyczne i badania doświadczalne anizotropowego tarcia suchego. Rozprawa doktorska, *Politechnika Łódzka*, (2020).

[9] S. V. Walker, R. I. Leine. Set-valued anisotropic dry friction laws: formulation, experimental verification and instability phenomenon. *Nonlinear Dynamics*, 96(2), 885–920 (2019).

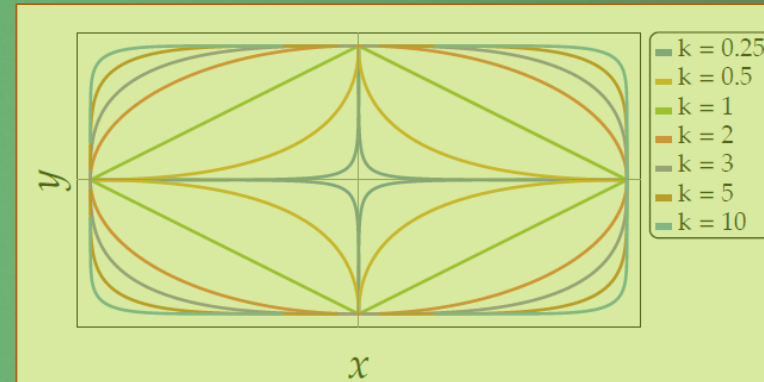
Modelowanie matematyczne



Modele fenomenologiczne



Modele empiryczne

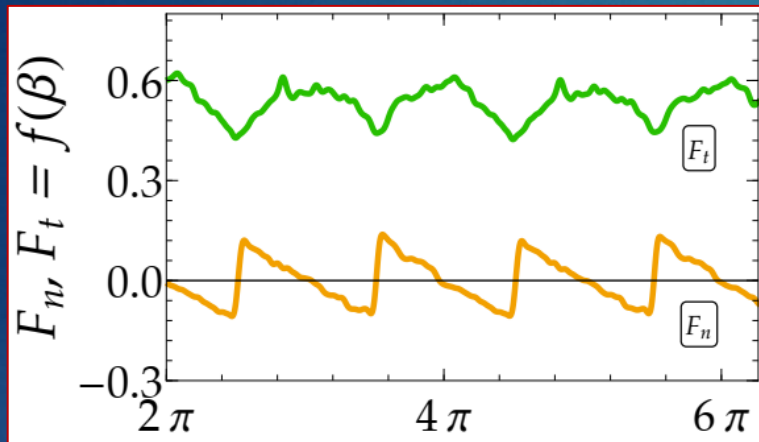


$$f(x, y) = \left| \frac{x}{c_1} \right|^k + \left| \frac{y}{c_2} \right|^k - 1 = 0$$

Modelowanie empiryczne

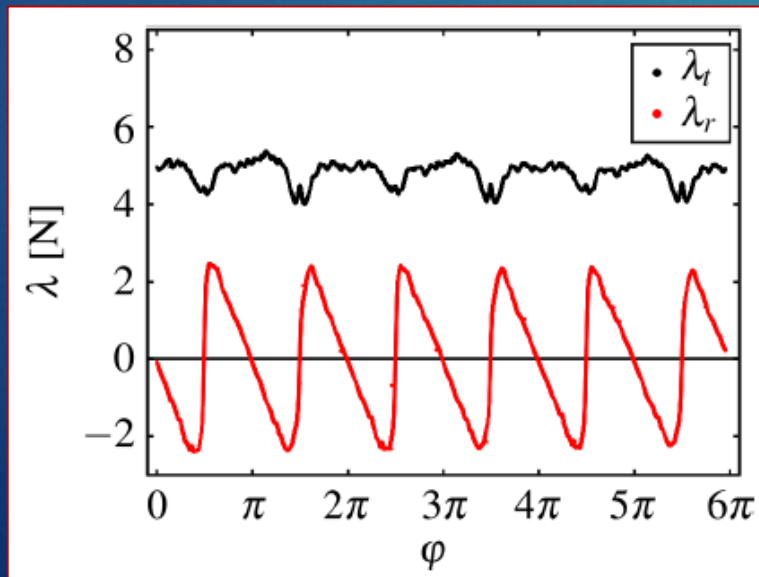
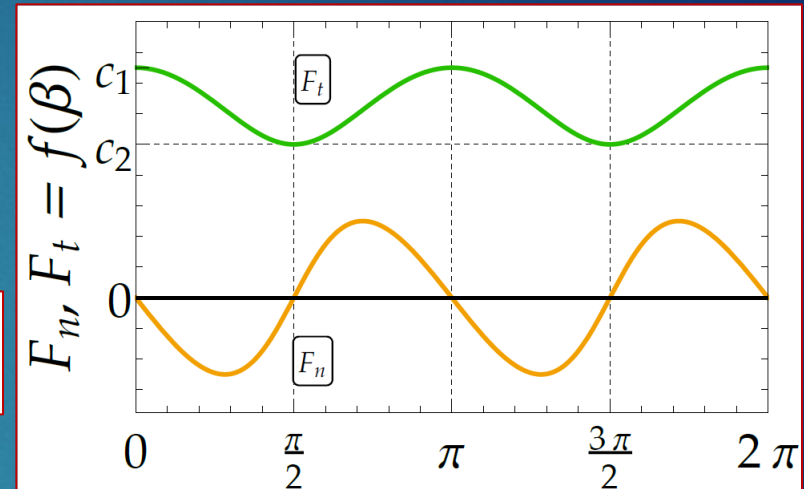


Elipsa



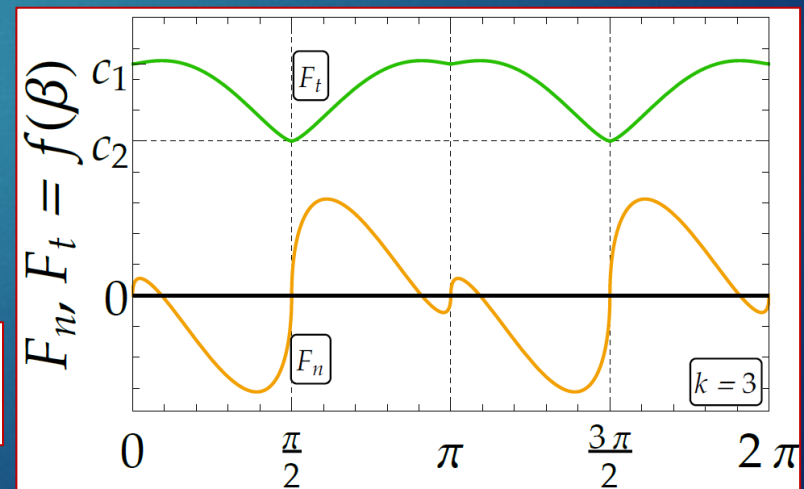
Elipsa

$$f(x, y) = \left(\frac{x}{c_1}\right)^2 + \left(\frac{y}{c_2}\right)^2 - 1 = 0$$



Superelipsa

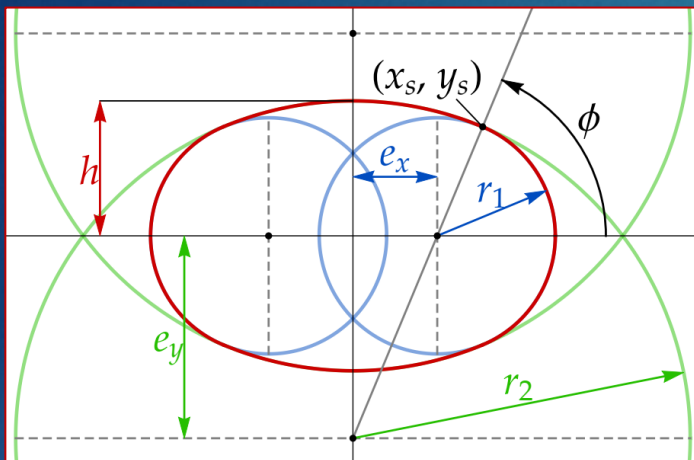
$$f(x, y) = \left|\frac{x}{c_1}\right|^k + \left|\frac{y}{c_2}\right|^k - 1 = 0$$





Modelowanie matematyczne

Owal

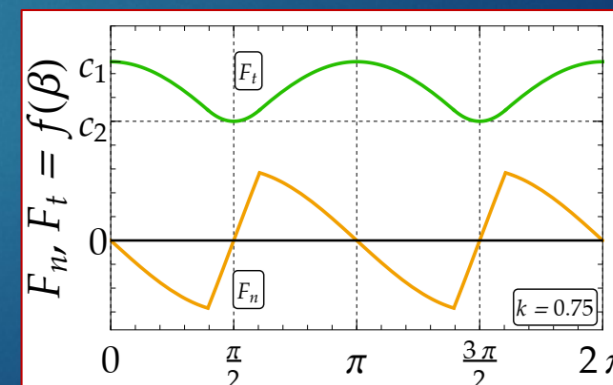
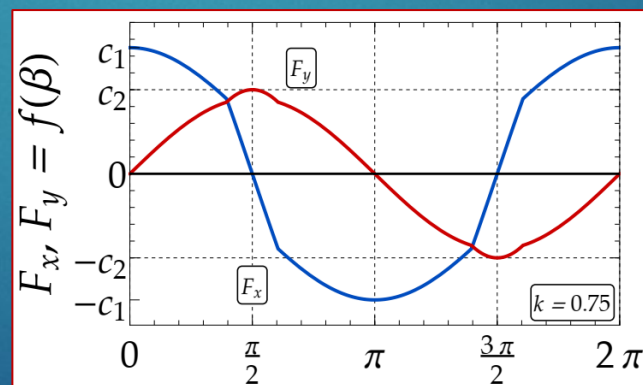
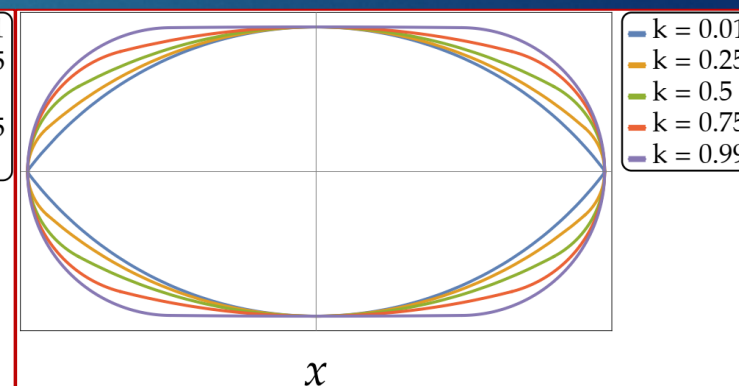
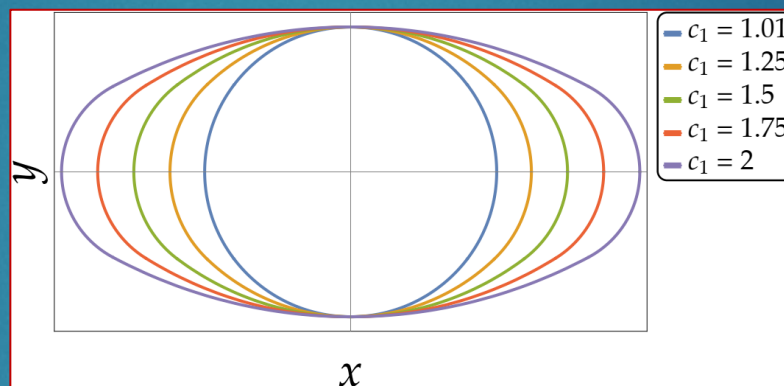


$$c_1 = r_1 + e_x, \quad c_2 = h, \quad k = \frac{c_1 (c_1 - c_2)}{c_2 e_x} - \frac{c_1 - c_2}{c_2}$$

$c_1 > c_2$ – długość półosi, $k \in (0,1)$ – współ. kształtu.

$$\begin{aligned} f_1(t) &= [r_1 \cos t + e_x, r_1 \sin t] \\ f_2(t) &= [r_2 \cos t, r_2 \sin t - e_y] \\ f_3(t) &= [r_1 \cos t - e_x, r_1 \sin t] \\ f_4(t) &= [r_2 \cos t, r_2 \sin t + e_y] \end{aligned}$$

$$f(t) = \begin{cases} f_1(t), & 0 < t \leq \phi \\ f_2(t), & -\phi < t \leq \pi - \phi \\ f_3(t), & \pi - \phi < t \leq \pi + \phi \\ f_4(t), & \pi + \phi < t \leq 2\pi - \phi \\ f_1(t), & 2\pi - \phi < t \leq 2\pi \end{cases}$$



Stowarzyszona reguła poślizgu

Identyfikacja danych pomiarowych

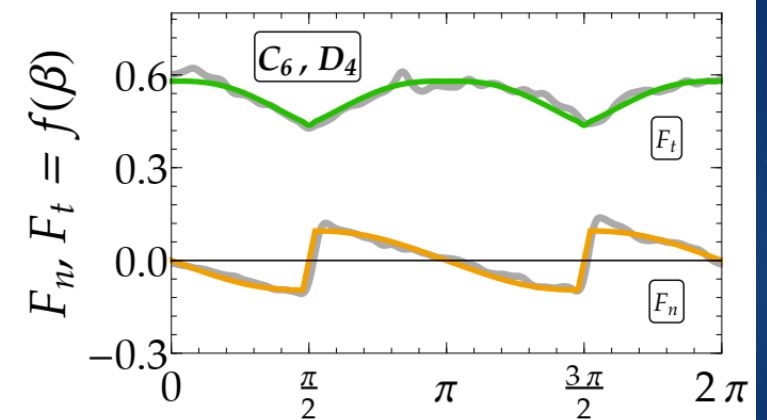
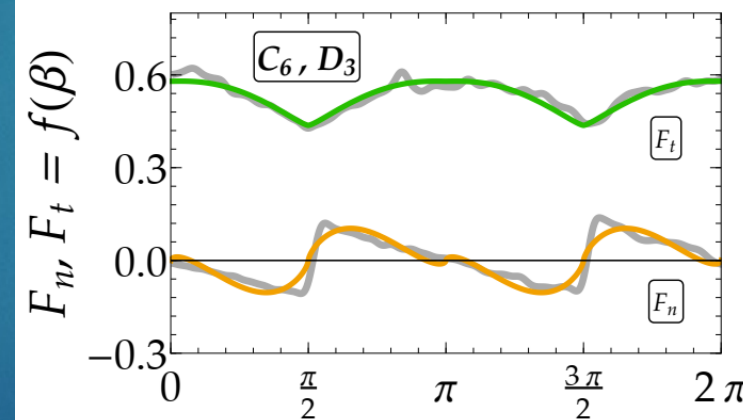
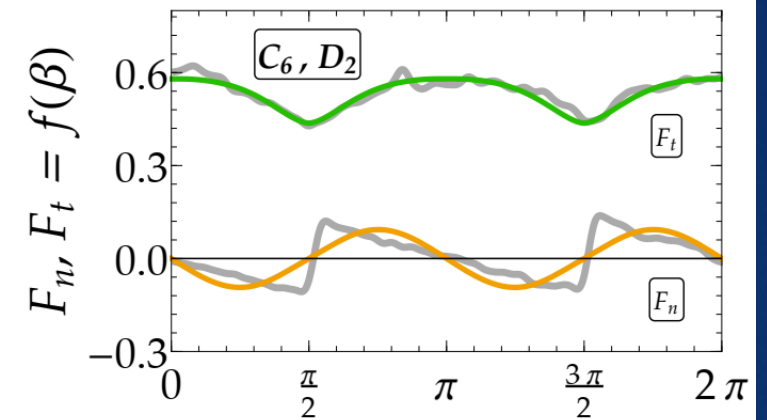
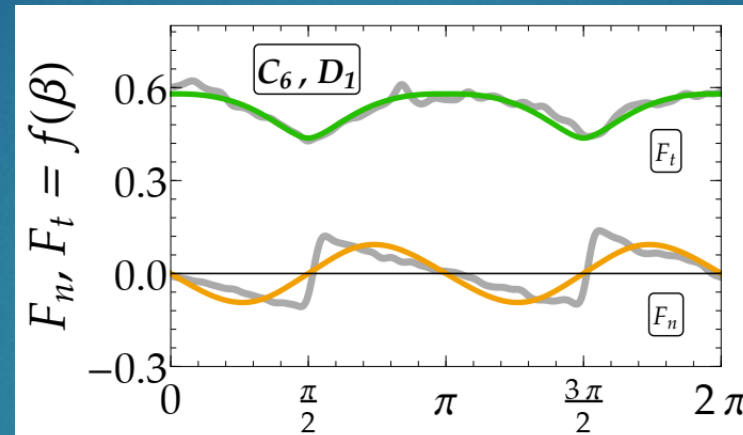
Reguła poślizgu – kąt $\varphi(\beta)$



- $\varphi = \alpha - \beta$
- β – kierunek poślizgu
- α – kierunek siły tarcia

D_1	Elipsa
D_2	$\cos(2\alpha)$
D_3	Superelipsa
D_4	Owal

	δ_D	k	k_p	R^2
D_1	1.189	–	0.858	0.735
D_2	1.181	–	0.841	0.707
D_3	1.187	2.27	0.954	0.911
D_4	1.205	0.923	0.911	0.954

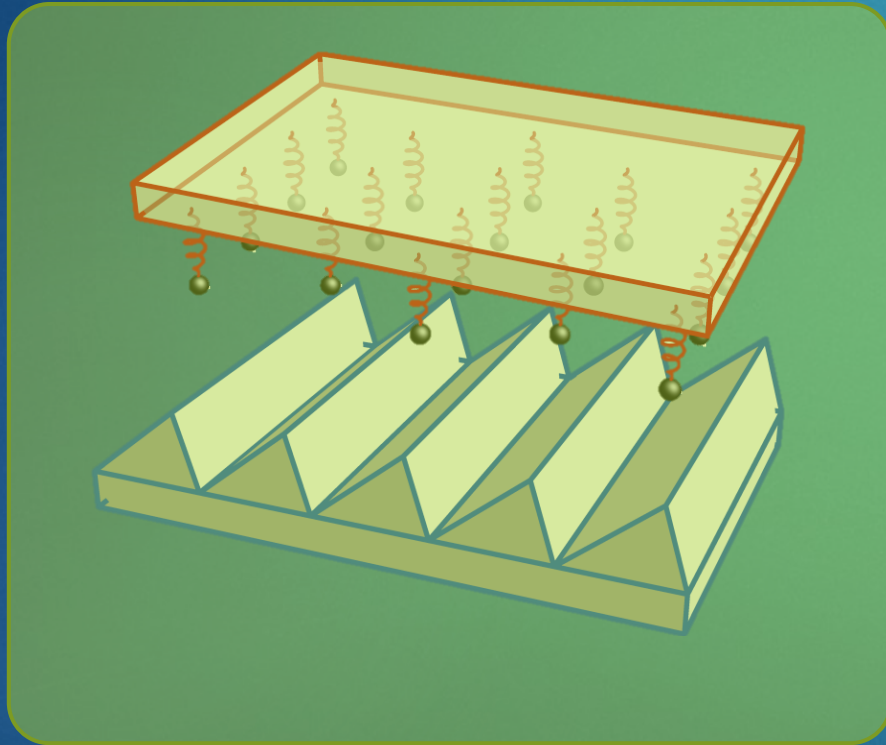


k_p – współczynnik korelacji Pearsona
 R^2 – współczynnik determinacji

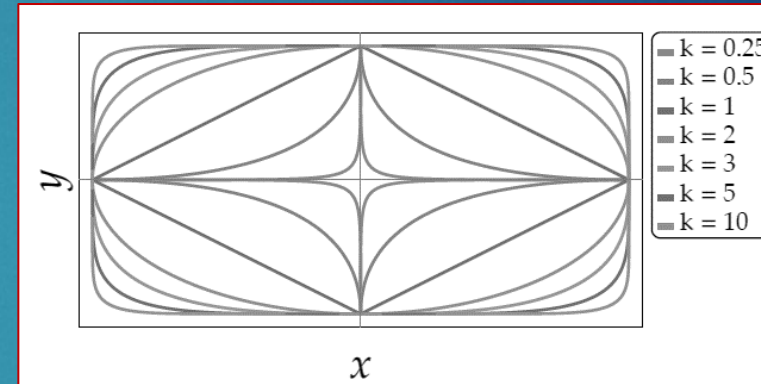
Modelowanie matematyczne



Modele fenomenologiczne



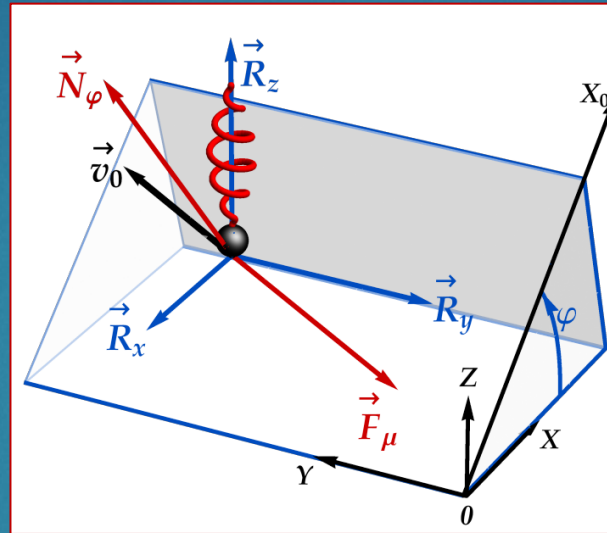
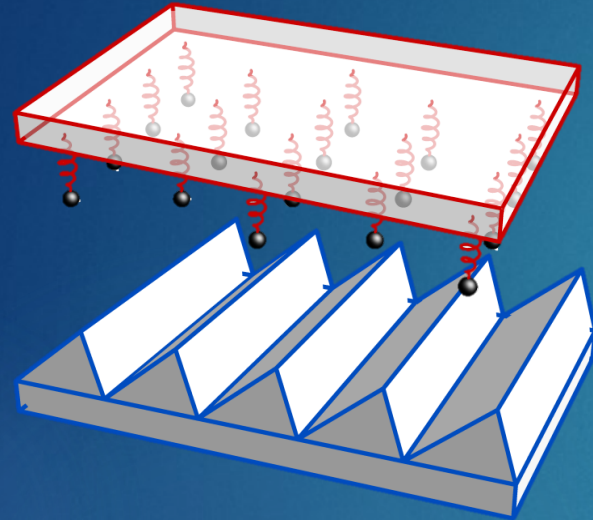
Modele empiryczne



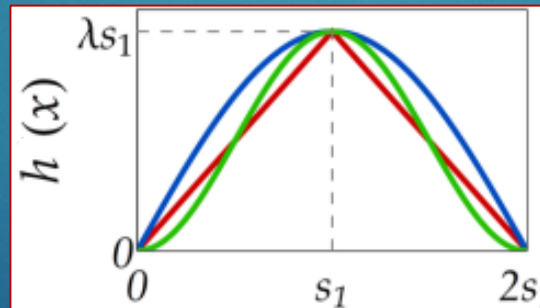
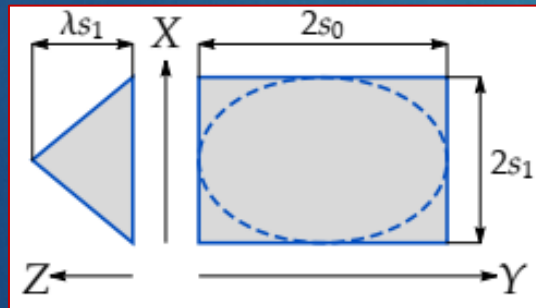
$$f(x, y) = \left| \frac{x}{c_1} \right|^k + \left| \frac{y}{c_2} \right|^k - 1 = 0$$




Badania charakterystyki statycznej

Model mikromechaniczny [10]



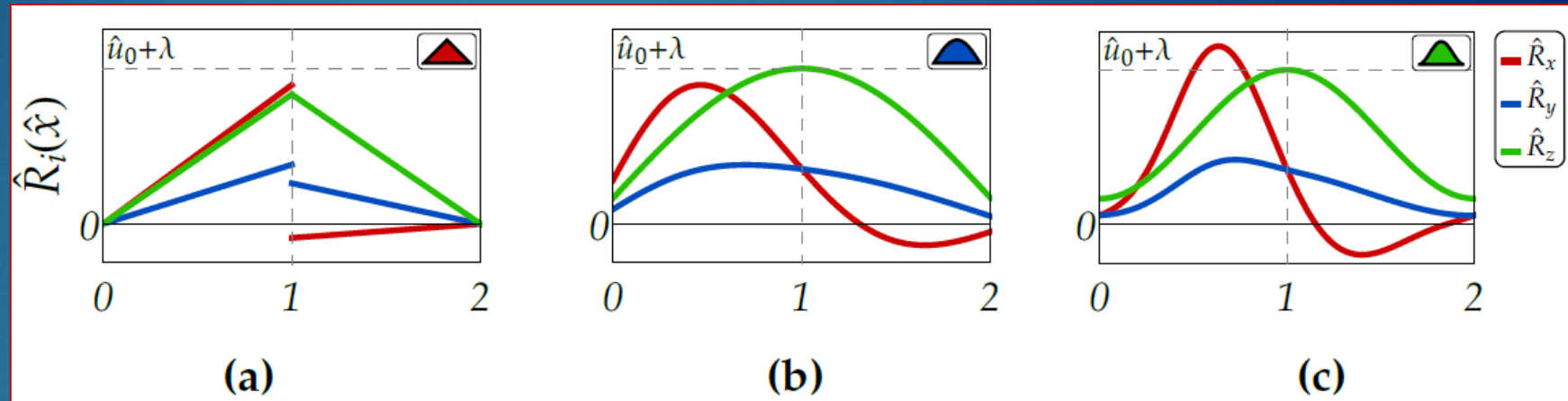
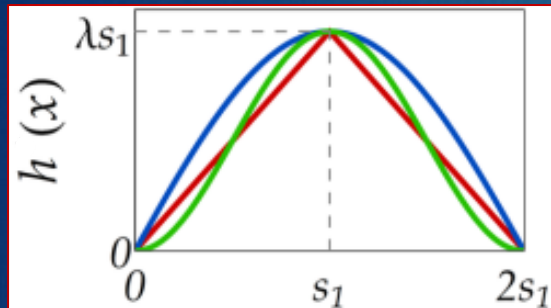
- ▶ Szczotka przyjęta jako lekka i podatna w kierunku Z (sztywność k_n)
- ▶ Kontakt szczotki z profilem to styk punktowy.
- ▶ Siła reakcji kontaktu składa się z reakcji normalnej N_φ i stycznej F_μ



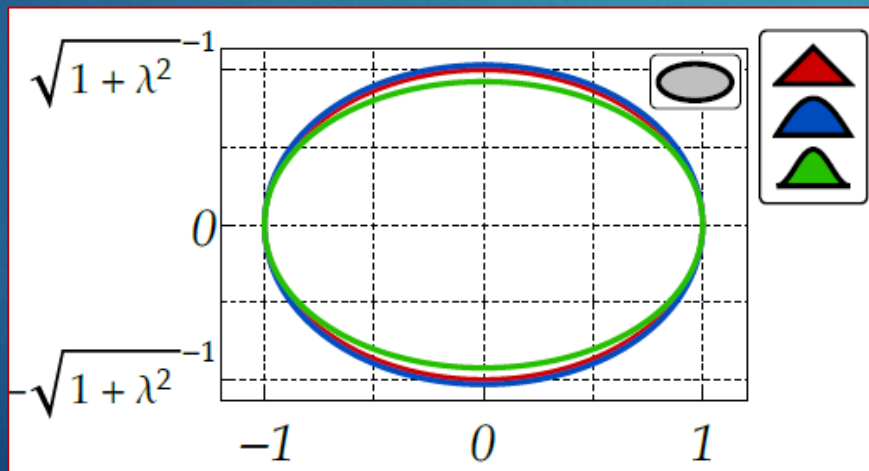
	$h(x)$
	$\begin{cases} \lambda x & 0 \leq x < s_1 \\ 2\lambda s_1 - \lambda x & s_1 \leq x \leq 2s_1 \end{cases}$
	$\lambda s_1 \sin\left(\frac{\pi x}{2s_1}\right)$
	$\frac{1}{2}\lambda s_1 \left(1 - \cos\left(\frac{\pi x}{s_1}\right)\right)$

Badania charakterystyki statycznej

Model mikromechaniczny. Wpływ profilu nierówności na potencjał poślizgu.



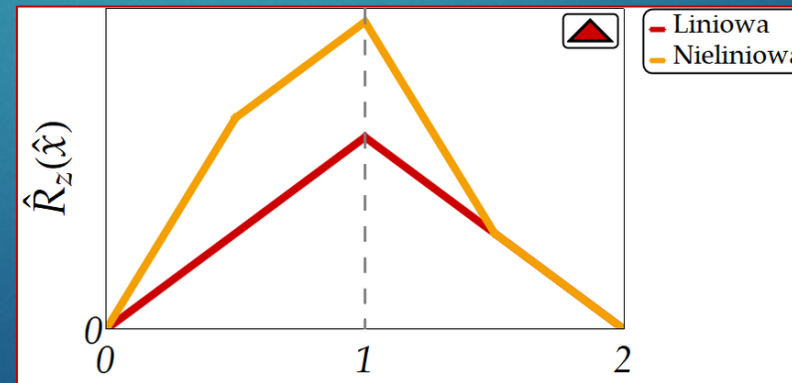
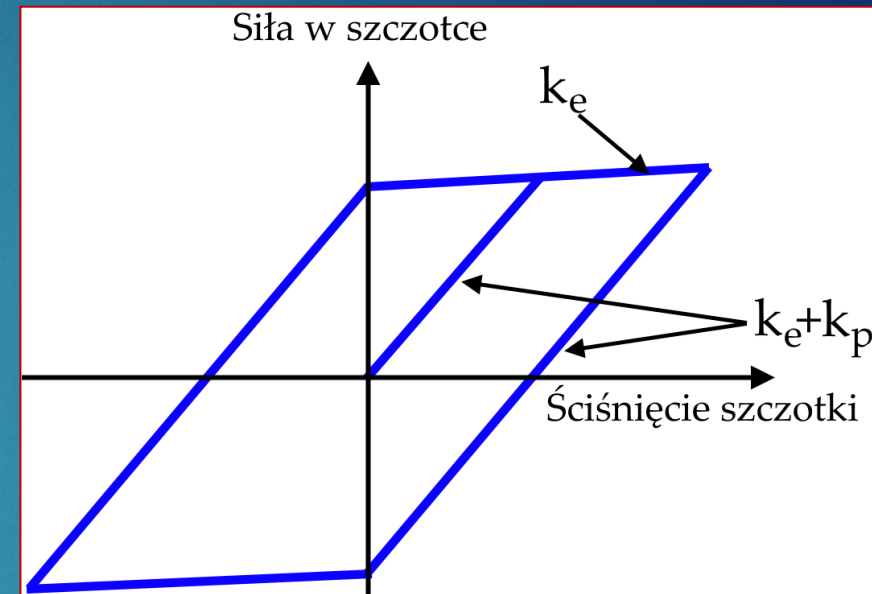
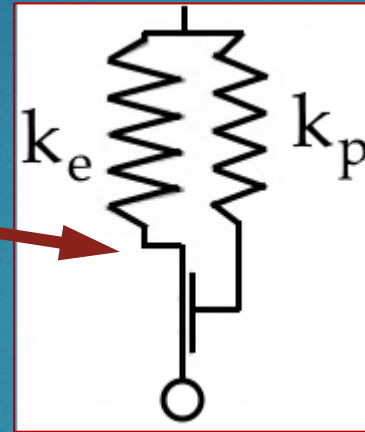
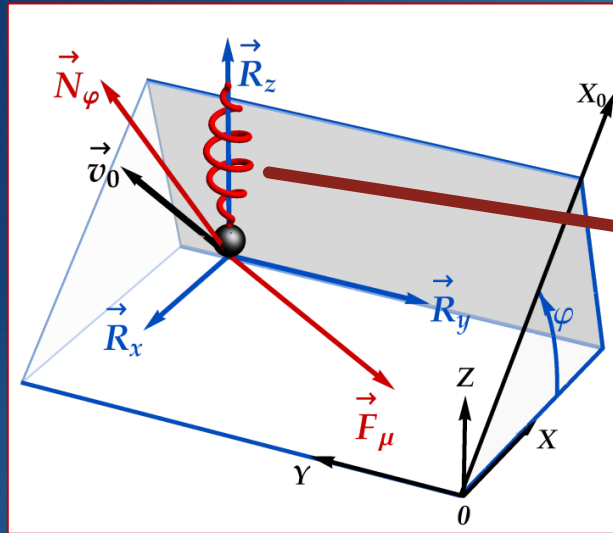
Wartości reakcji $R_i(x)$ dla profilu trójkątnego (a), półfali sinusoidy (b) i odwróconej cosinusoidy (c) (kierunek poślizgu $\beta = \frac{\pi}{4}$)



Potencjał poślizgu otrzymany dla różnych profili nierówności modelu mikromechanicznego.

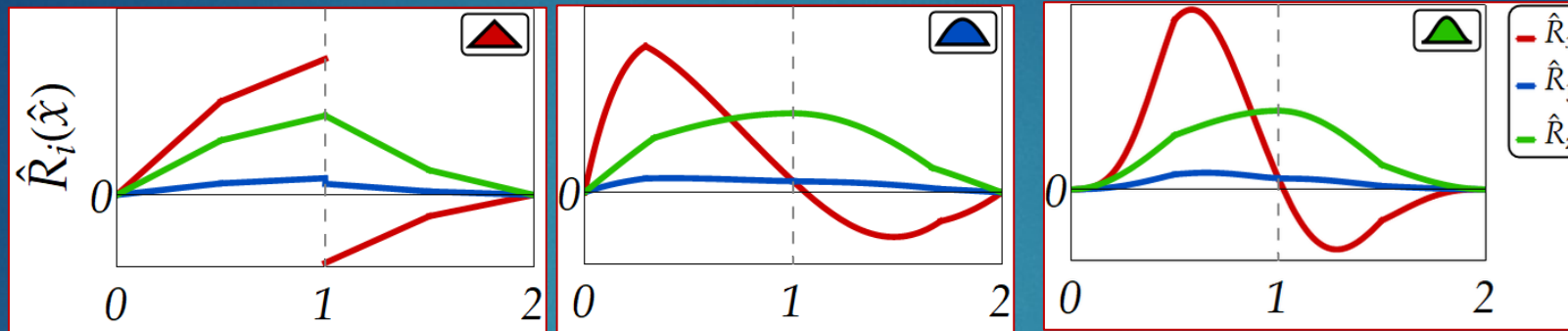
Badania charakterystyki statycznej

Model mikromechaniczny. Szczotka sprężysto-plastyczna.

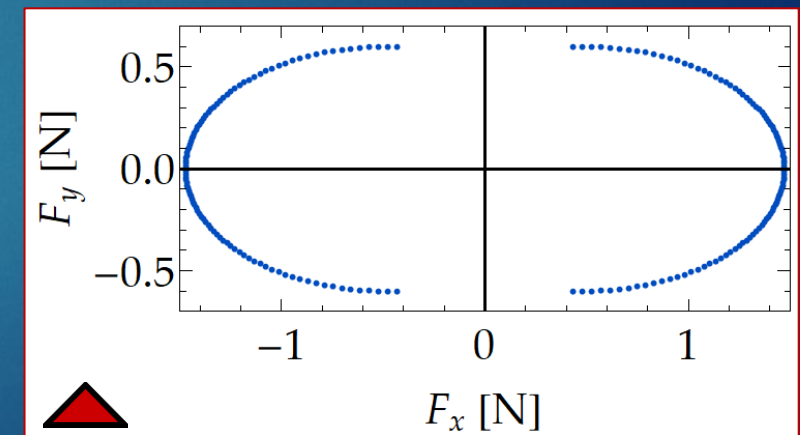
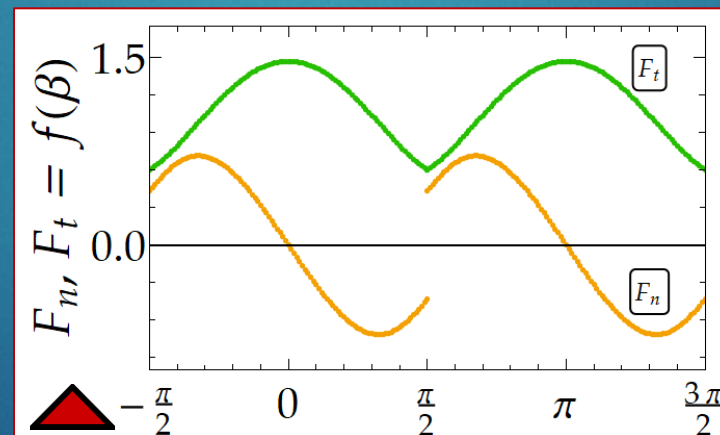
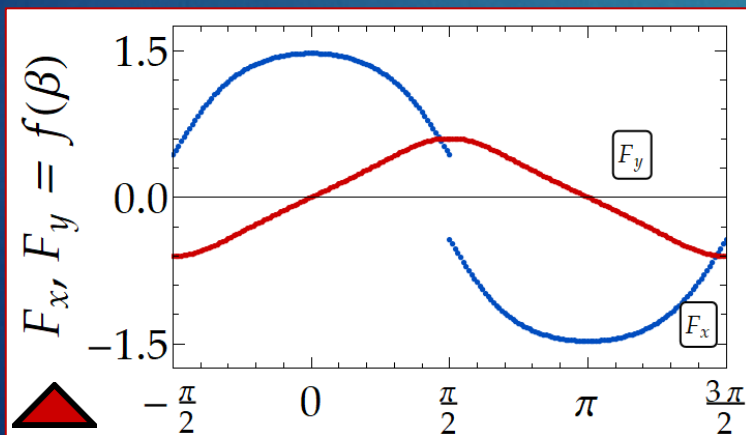


Badania charakterystyki statycznej

Model mikromechaniczny. Szczotka sprężysto-plastyczna.



Wartości reakcji $R_i(x)$ dla profilu trójkątnego (a), pół-fali sinusoidy (b) i odwróconej cosinusoidy (c) (kierunek poślizgu $\beta = \frac{\pi}{4}$)

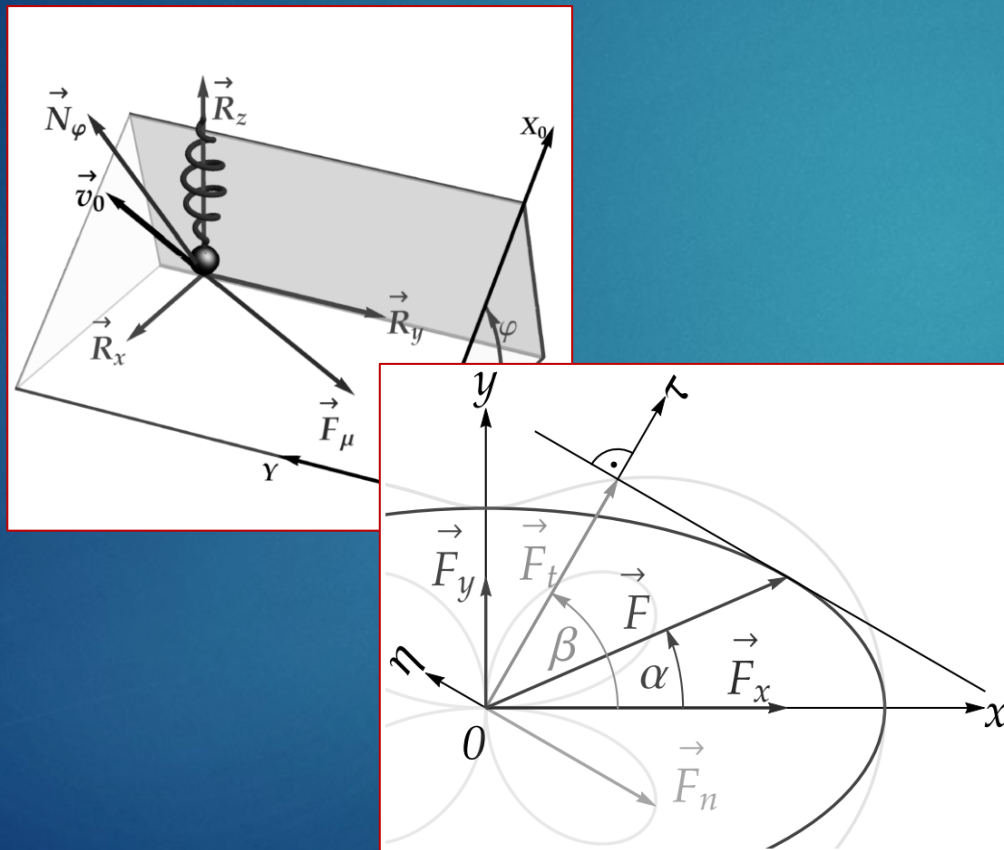


Wartości reakcji $R_i(x)$ dla profilu trójkątnego (a), półfali sinusoidy (b) i odwróconej cosinusoidy (c) ($\lambda = 0.5, \beta = \frac{\pi}{4}, \mu = 0.5$)

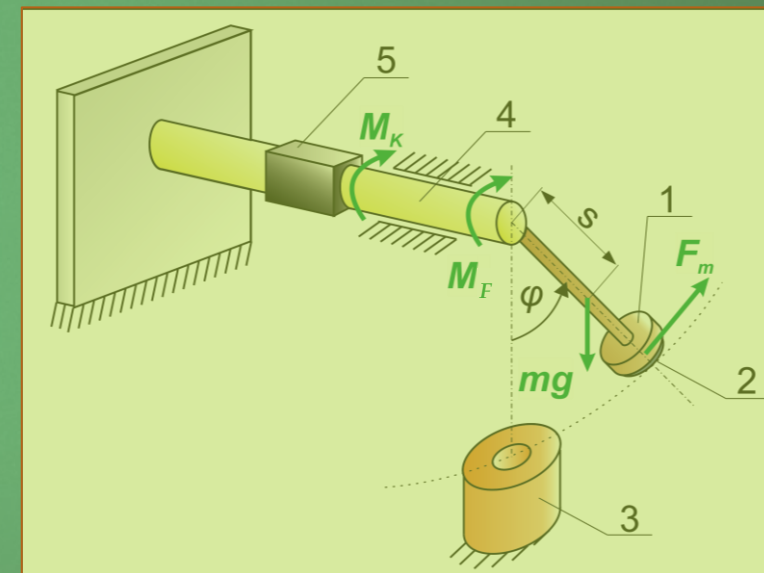
Plan prezentacji



Anizotropowe tarcie suche

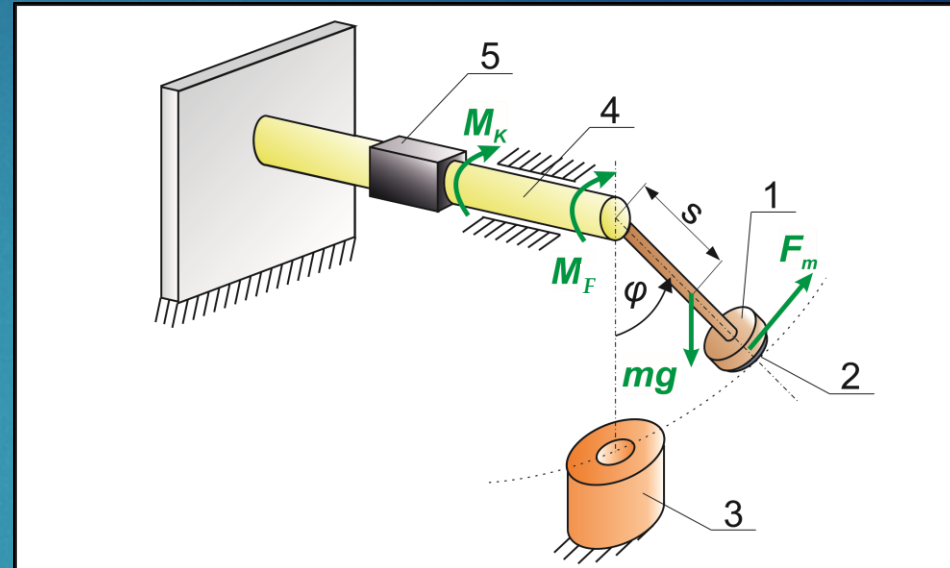
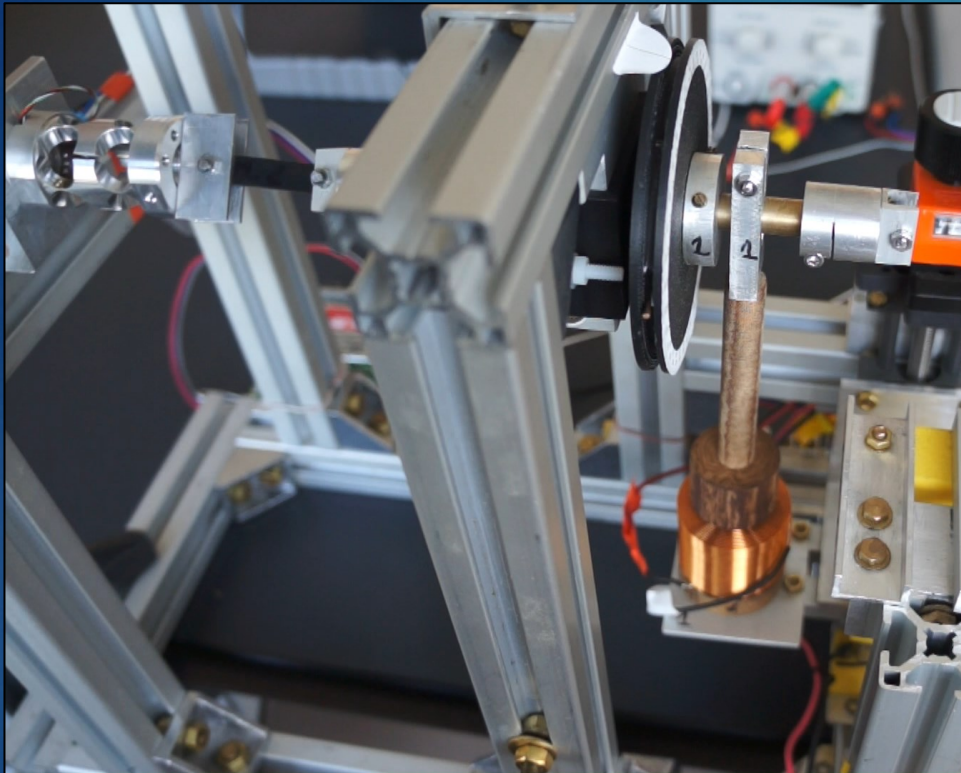


Wahadło magnetyczne

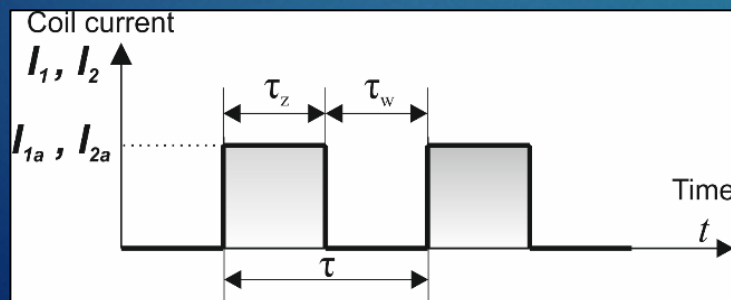


Wahadło magnetyczne

Stanowisko badawcze i modelowanie



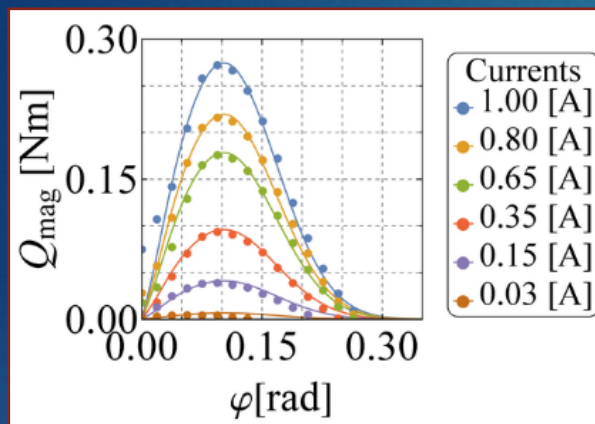
1-wahadło fizyczne, 2-magnes neodymowy, 3-cewka, 4-oś, 5-elastyczny łącznik.



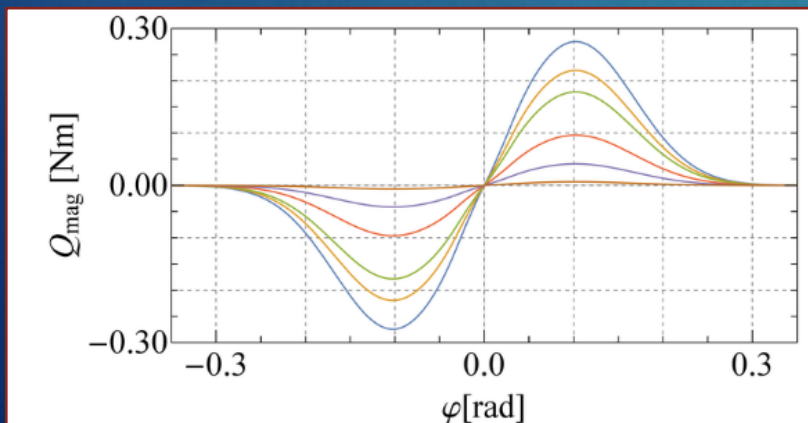
$$I\ddot{\varphi} = -mgs \sin \varphi - M_K(\varphi, \dot{\varphi}) - M_F(\dot{\varphi}) + Q_{mag}(\varphi, t),$$

[11] Wijata, A., Polczyński, K., i Awrejcewicz, J. Theoretical and numerical analysis of regular one-side oscillations in a single pendulum system driven by a magnetic field, *Mechanical Systems and Signal Processing*, 150 (107229) (2021)

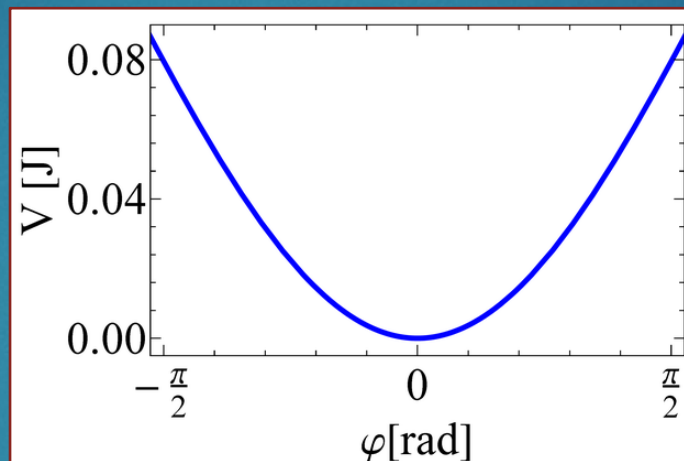
Wahadło magnetyczne



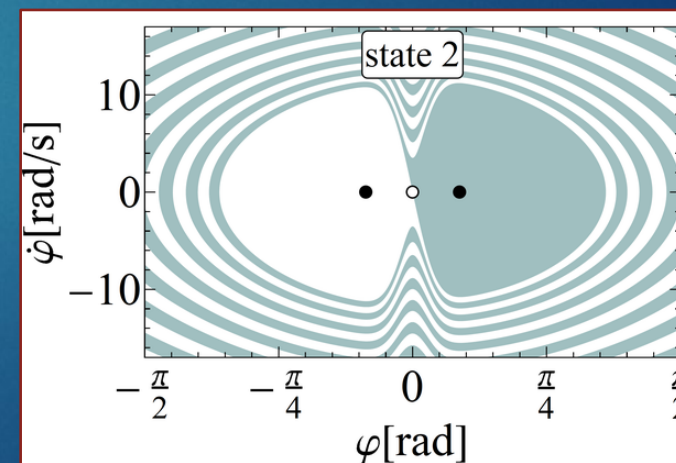
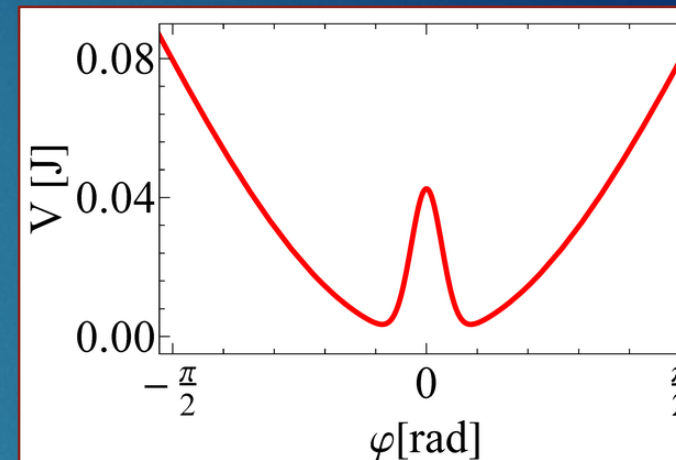
$$Q_{mag}(\varphi, t) = a(i(t)) \exp\left(\frac{-\varphi^2}{c_1}\right) \varphi$$



Wahadło grawitacyjne

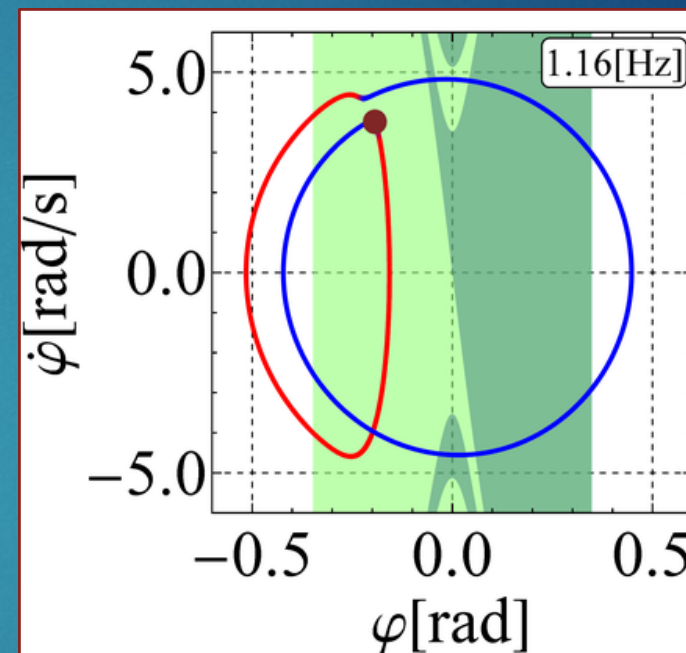
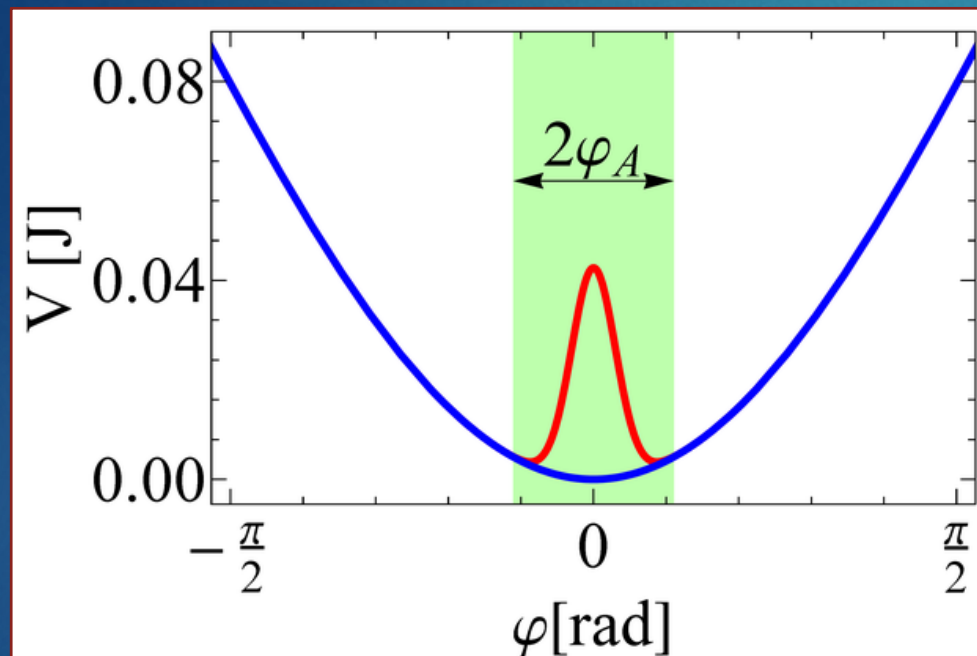


Wahadło magnetyczne



Wahadło magnetyczne

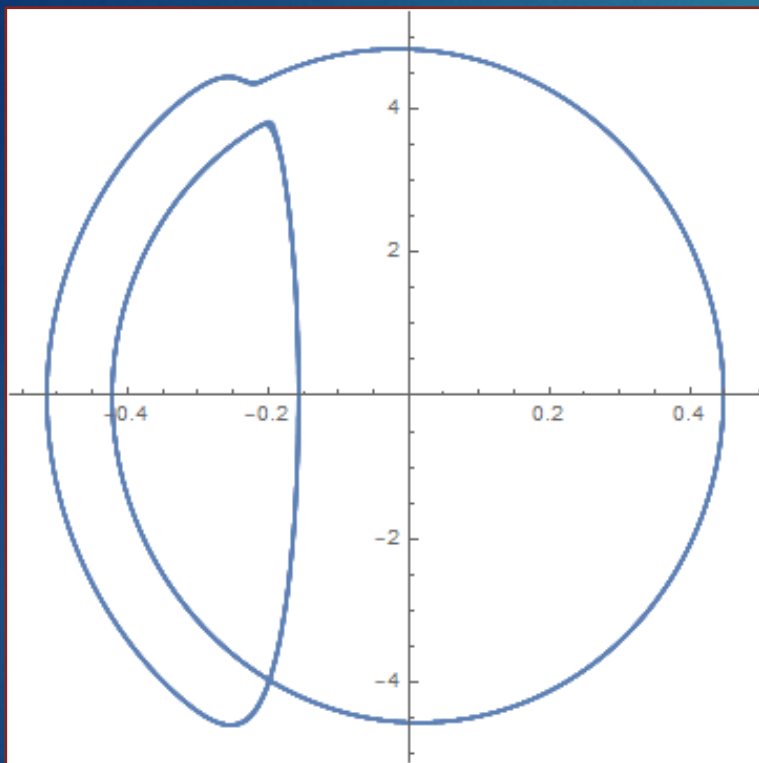
Prezentacja wyników





Prezentacja wyników

Podstawowy wykres – ustawienia automatyczne programu Mathematica

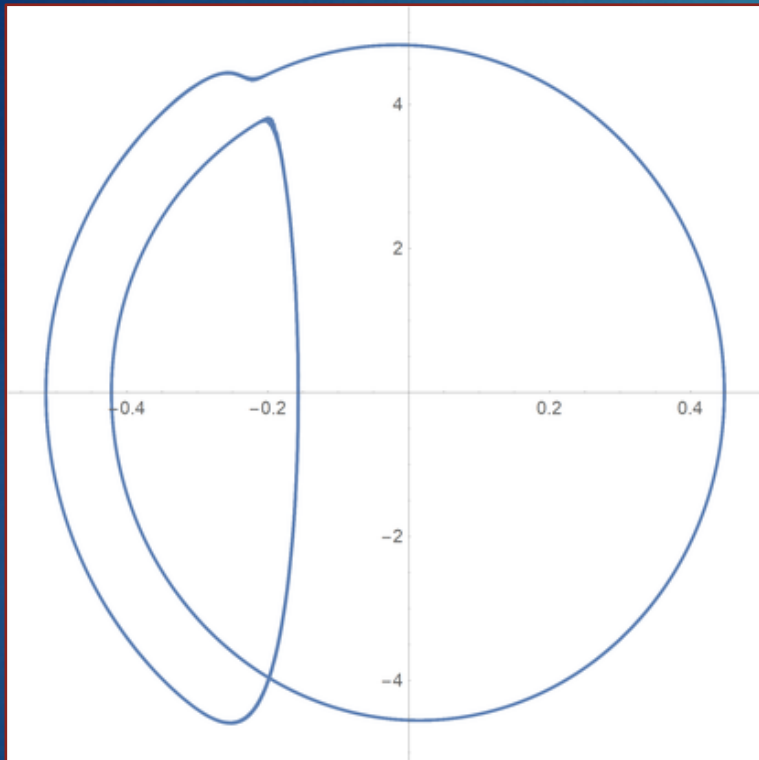


```
p116A = ParametricPlot[{ $\phi_1[t]$ ,  $\phi_1'[t]$ } /. solPS,  
  {t, t0, t1}, AspectRatio  $\rightarrow$  1]
```

```
file := "1_16Hz_A_";  
format := ".tiff";  
fileName := StringJoin[{file, format}]  
  
Export[NotebookDirectory[] <> "\\Figures\\" <>  
  fileName, p116A]
```

Prezentacja wyników

Rozdzielczość wykresu - dpi (ang. *dots per inch*)



```
p116A = ParametricPlot[{ $\phi_1[t]$ ,  $\phi_1'[t]$ } /. solPS,  
  {t, t0, t1}, AspectRatio -> 1]
```

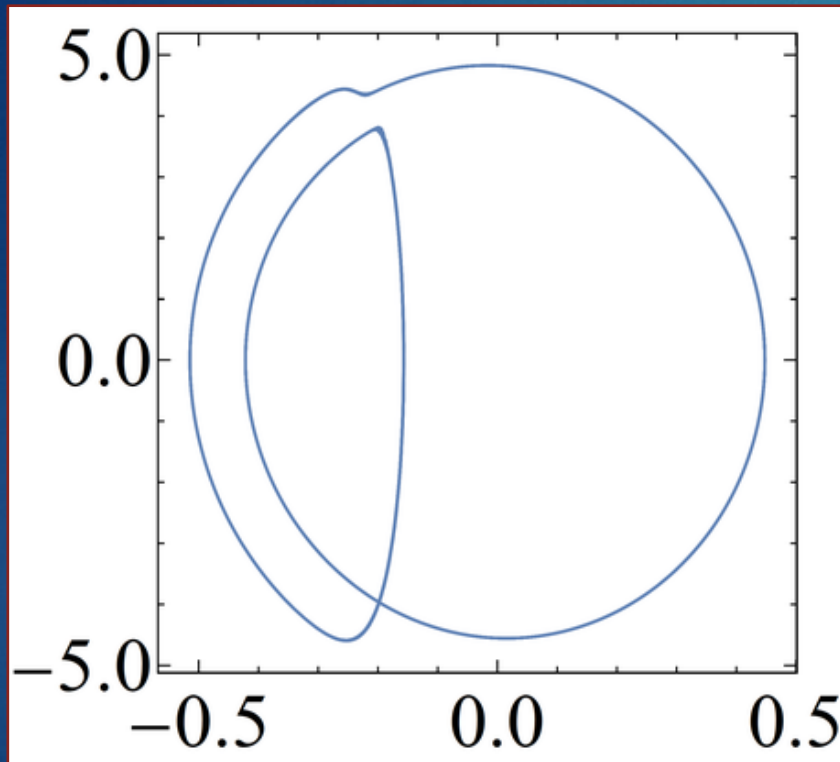
```
res = 300; (*rozdzielczość - DPI*)  
cm = res / 2.54; (*centymetr w zaleznosci od DPI*)  
h = 3.5 cm; (*wysokość*)  
k = res / 72; (*wspolczynnik skalowania  
np: do rozmiaru czcionek*)
```

```
file := "1_16Hz_B_" <> ToString[res] <> "DPI";  
format := ".tiff";  
fileName := StringJoin[{file, format}]
```

```
Export[NotebookDirectory[] <> "\\Figures\\" <>  
  fileName, p116B,  
  ImageSize -> {Automatic, h}, (*W x H*)  
  ImageResolution -> res (*dpi do rasteryzacji*)  
]
```

Prezentacja wyników

Ramka i skalowanie czcionek



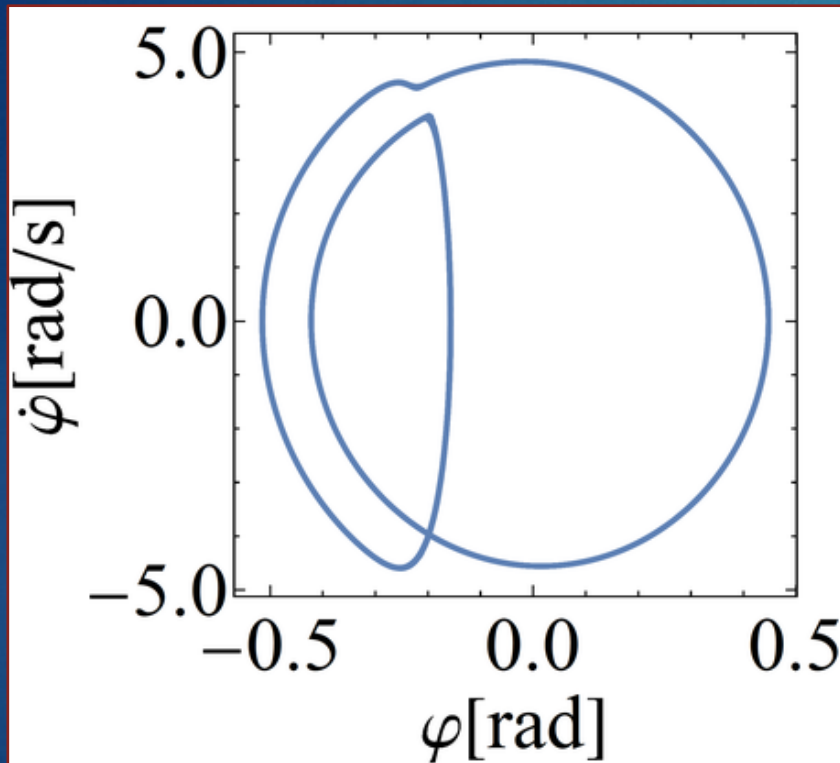
```
p116C = ParametricPlot[ $\{\phi_1[t], \phi_1'[t]\}$  /. solPS, {t, t0, t1},  
  AspectRatio → 1,  
  ImageSize → {Automatic, h},  
  Frame → True,  
  FrameStyle → Black,  
  FrameTicks → ticksGen[Range[-1, 1, 0.5], Range[-1, 1, 0.1],  
    Range[-5., 5., 5], Range[-5., 5., 1], 0.02, 0.01, 1, 1],  
  Axes → {False, False}  
]
```

```
ticksGen[xMaj_, xMin_, yMaj_, yMin_, hMaj_, hMin_, dX_, dY_] := (  
  xMajor =  
    {#, Style[NumberForm[#, {3, dX}], ticksStyle], {hMaj, 0}} & /@ xMaj;  
  xMajor2 = {#, Null, {hMaj, 0}} & /@ xMaj;  
  xMinor = {#, Null, {hMin, 0}} & /@ xMin;  
  
  yMajor =  
    {#, Style[NumberForm[#, {3, dY}], ticksStyle], {hMaj, 0}} & /@ yMaj;  
  yMajor2 = {#, Null, {hMaj, 0}} & /@ yMaj;  
  yMinor = {#, Null, {hMin, 0}} & /@ yMin;  
  
  {{Join[yMajor, yMinor], Join[yMajor2, yMinor]},  
   {Join[xMinor, xMajor], Join[xMinor, xMajor2]}}  
)
```

```
ticksStyle := {FontSize → 10 k, FontFamily → "Times", Black};  
labelStyle := {FontFamily → "Times", FontSize → 10 k, Black};  
labelStyleSym := {FontFamily → (*"Symbol"*) "Times",  
  FontSize → 10 k, Black};  
textStyle := {FontFamily → "Times", FontSize → 8 k, Black};
```

Prezentacja wyników

Opis osi (ramki) i grubość linii

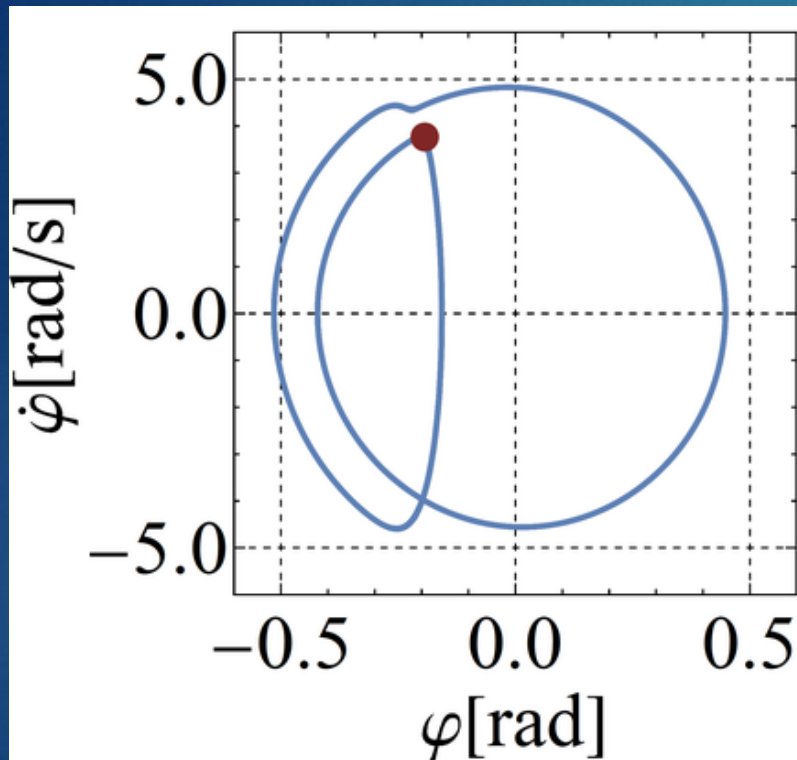


```
p116D = ParametricPlot[{phi_1[t], phi_1'[t]} /. solPS, {t, t0, t1},  
  AspectRatio -> 1,  
  ImageSize -> {Automatic, h},  
  Frame -> True,  
  FrameStyle -> Black,  
  FrameTicks -> ticksGen[Range[-1, 1, 0.5], Range[-1, 1, 0.1],  
    Range[-5., 5., 5], Range[-5., 5., 1], 0.02, 0.01, 1, 1],  
  Axes -> {False, False},  
  
  PlotStyle -> {Thickness[0.01]},  
  FrameLabel ->  
    {Row[{Style["phi", labelStyle], Style["[rad]", labelStyle]}],  
     Row[{Style["phi-dot", labelStyle], Style["[rad/s]", labelStyle]}}]}
```

]

Prezentacja wyników

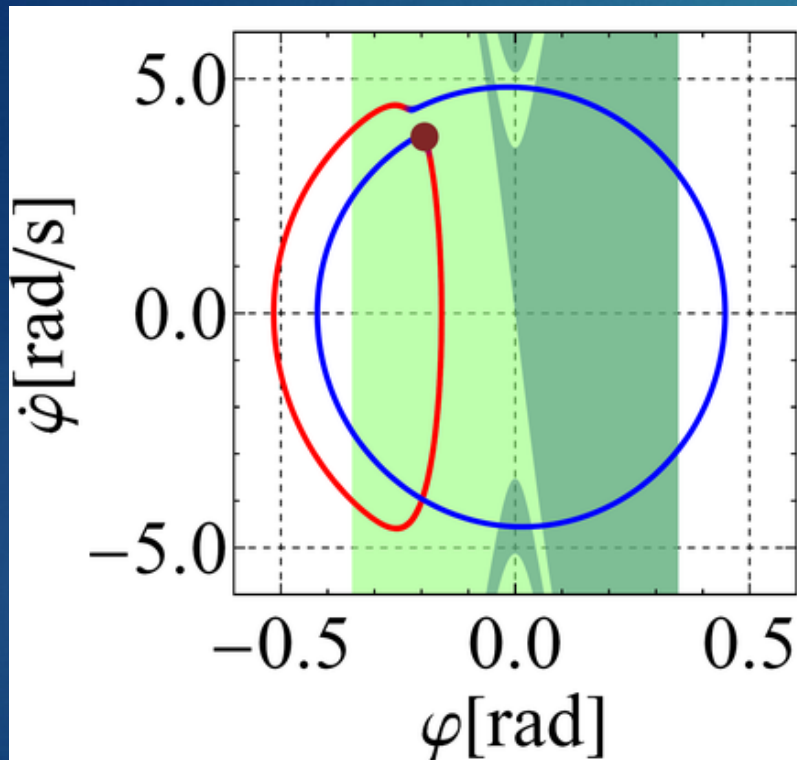
Zakres wykresu, siatka i wstawianie punktu



```
p116E = ParametricPlot[{phi_1[t], phi_1'[t]} /. solPS, {t, t0, t1},
  AspectRatio -> 1,
  ImageSize -> {Automatic, h},
  Frame -> True,
  FrameStyle -> Black,
  FrameTicks -> ticksGen[Range[-1, 1, 0.5], Range[-1, 1, 0.1],
    Range[-5., 5., 5], Range[-5., 5., 1], 0.02, 0.01, 1, 1],
  Axes -> {False, False},
  PlotStyle -> {Thickness[0.01]},
  FrameLabel ->
    {Row[{Style["phi", labelStyle], Style["[rad]", labelStyle]}],
     Row[{Style["phi-dot", labelStyle],
           Style["[rad/s]", labelStyle]}]},
  |
  PlotRange -> {{-0.6, 0.6}, {-6, 6}},
  GridLines -> {Range[-1, 1, 0.5], Range[-5, 5, 5]},
  GridLinesStyle -> Directive[Black, Dashed],
  Epilog -> {
    {PointSize[0.05], Hue[0, 0.7, 0.5, 1],
     Point[dataPS[[All, 1 ;; 2]]]}
  }
]
```

Prezentacja wyników

Zielony obszar (baseny przyciągania) i zmienny kolor linii



```
p116F = ParametricPlot[{phi_1[t], phi_1'[t]} /. solPS, {t, t0, t1},
```

Prolog →

```
{ {Hue[0.3, 0.5, 1, 0.64], Rectangle[{-phi_A, -20}, {phi_A, 20}]},  
  {Hue[0.5, 0.5, 0.5, 0.5],  
    Polygon[boALine]}
```

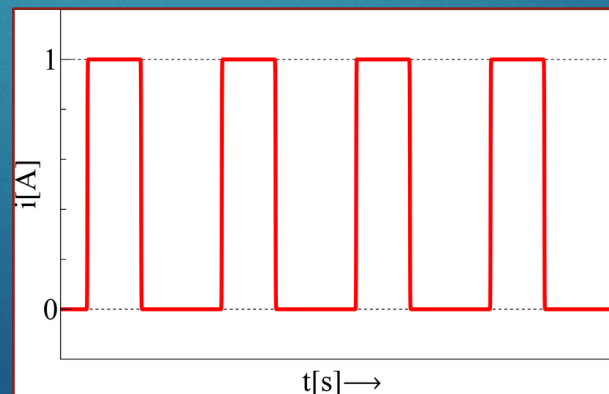
```
},
```

ColorFunction → Function[{x, y, u},

```
If[i_1[u, w, f] > .5, Red, Blue]],
```

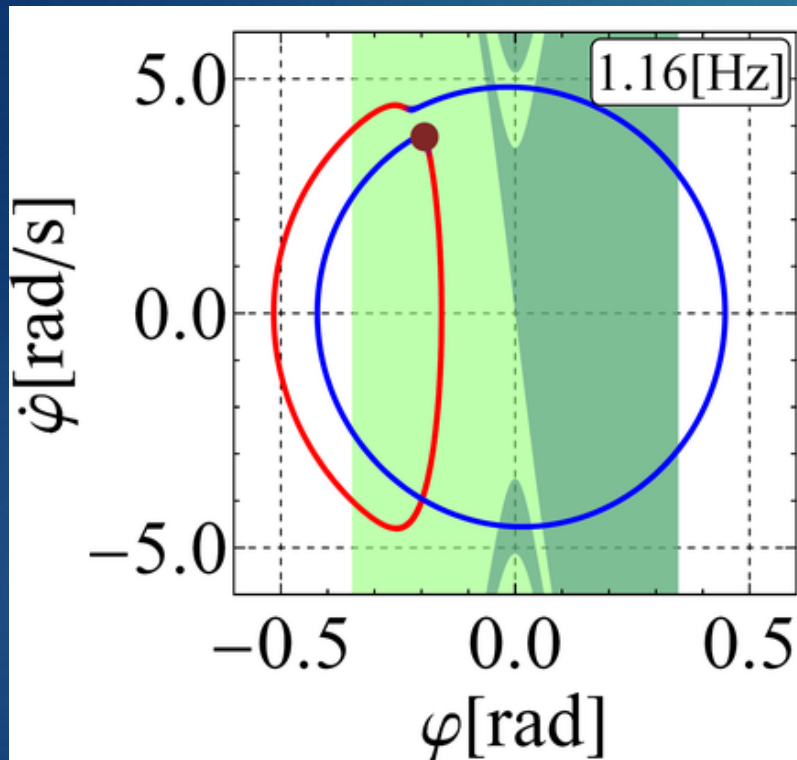
ColorFunctionScaling → False,

```
(...)
```



Prezentacja wyników

Ramka z dodatkowymi informacjami



```
p116G = ParametricPlot[{phi_1[t], phi_1'[t]} /. solPS, {t, t0, t1},
```

Prolog →

```
{ {Hue[0.3, 0.5, 1, 0.64], Rectangle[{-phi_A, -20}, {phi_A, 20}]},  
  {Hue[0.5, 0.5, 0.5, 0.5],  
    Polygon[boALine]}  
},
```

(...)

×

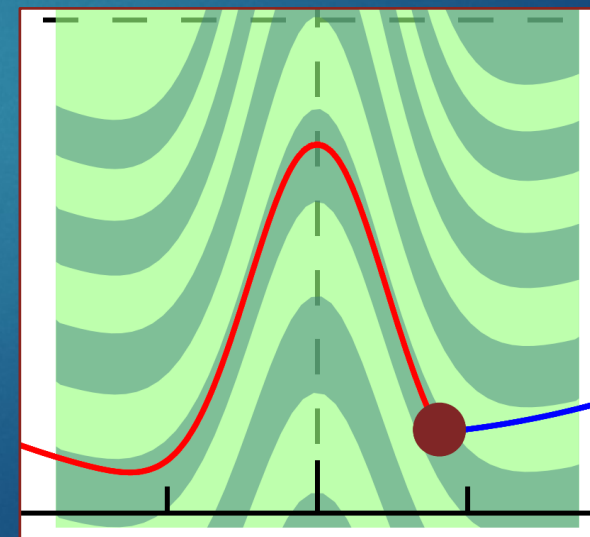
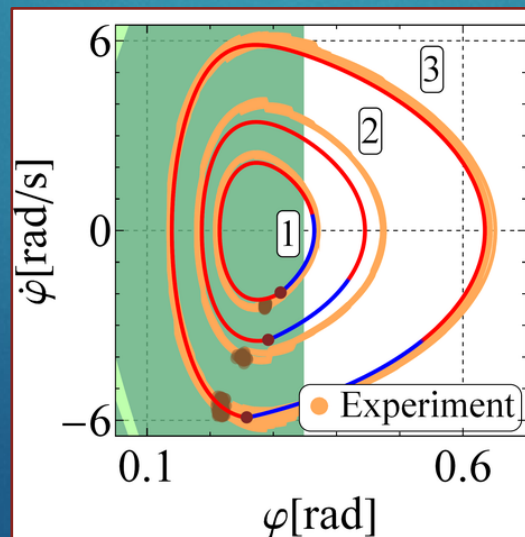
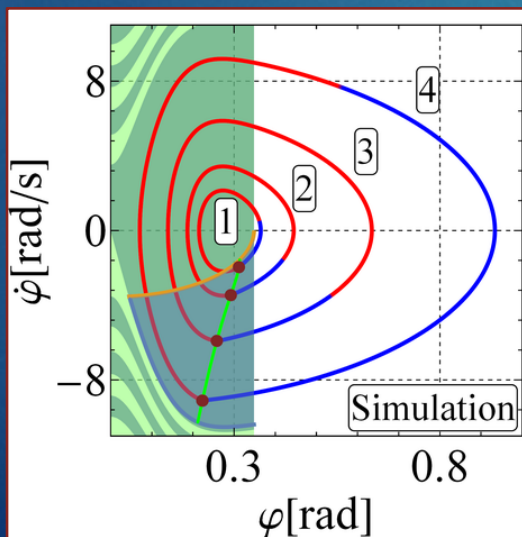
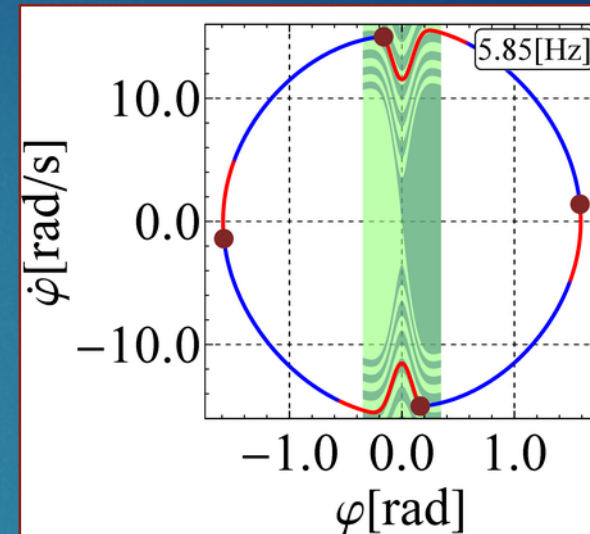
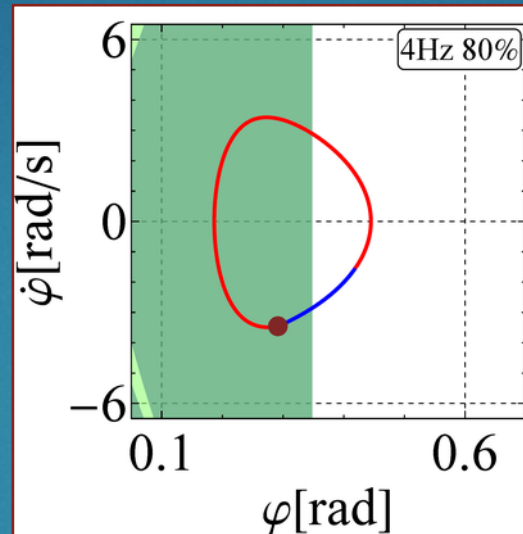
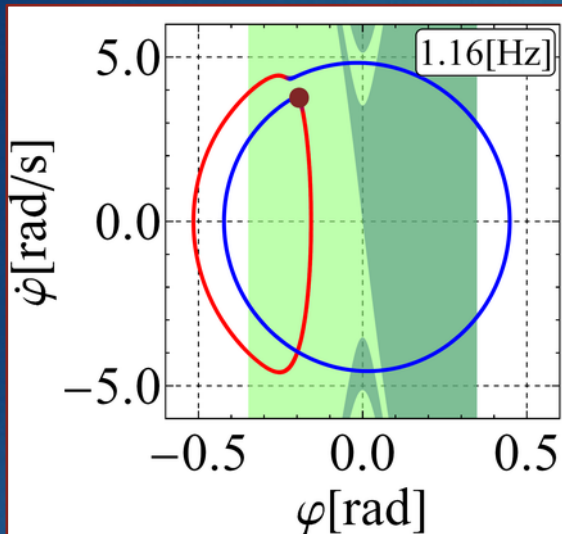
Epilog → {

```
{PointSize[0.05], Hue[0, 0.7, 0.5, 1],  
  Point[dataPS[[All, 1 ;; 2]]]},
```

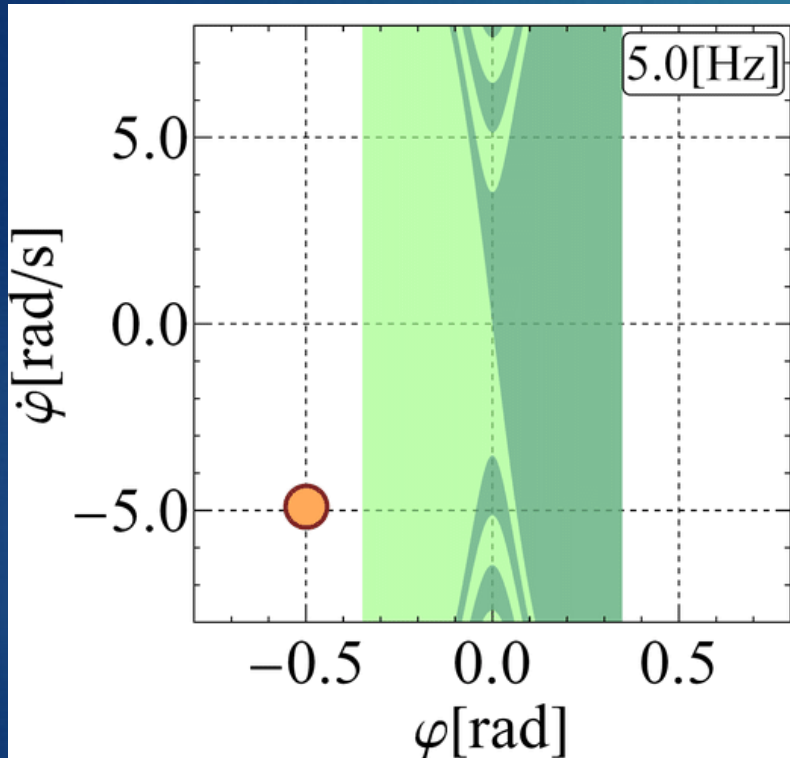
```
{Inset[Framed[Text[Style["1.16[Hz]", textStyle]],  
  FrameMargins → 1, RoundingRadius → 5,  
  FrameStyle → {Black, Thickness[1.5]},  
  Background → White], Scaled[{0.99, 0.99}],  
  {Right, Top}]}  
}
```

```
]
```

Prezentacja wyników



Prezentacja wyników – animacja .gif



```
plotAnim[ zθ_ ] := ParametricPlot[ {φ1[t], φ1'[t]} /. solPS,  
  {t, t0, Min[ zθ, t1] (*t1*)}, (...)
```

```
(*dla gifa fps:{100,50,25,20,10,5,4,2,1},  
ale IE np: wyswietla max 16... spowlania gify*)
```

```
frameRate = 50; (*klatek na sekundę*)  
slowMo = 0.25; (*Procent "rzeczywistej szybkości"*)  
czasGifa = 10; (*s*)  
czasSym = slowMo * czasGifa; (*Czas symulacji*)  
dt = slowMo * (1 / frameRate); (*Czas trwania jednej klatki*)
```

```
tabGif = plotAnim[#] & /@ Range[t0 + 0.001, t0 + czasSym - 0.001, dt];
```

```
file := "5_00Hz_" <> ToString[h] <> "px_" <> ToString[frameRate] <>  
  "fps_" <> ToString[slowMo] <> "slowMo_";
```

```
format := ".gif";
```

```
fileName := StringJoin[{file, format}]
```

```
Export[NotebookDirectory[] <> "\\Animacje\\" <> fileName, tabGif,
```

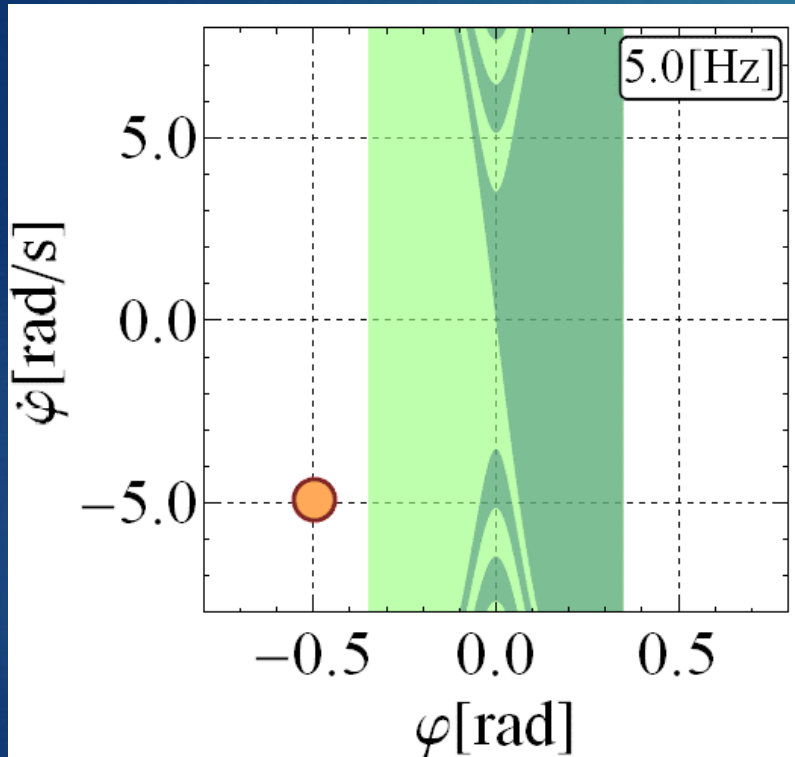
```
"DisplayDuration" → dt,
```

```
"AnimationRepetitions" → Infinity,
```

```
ImageSize → {Automatic, h}, (*W x H*)
```

```
ImageResolution → res ]
```

Prezentacja wyników – filmik .avi



```
plotAnim[ z0_ ] := ParametricPlot[ {phi_1[t], phi_1'[t]} /. solPS,  
  {t, t0, Min[ z0, t1] (*t1*)}, (...)
```

```
(*dla gifa fps:{100,50,25,20,10,5,4,2,1},  
ale IE np: wyswietla max 16... spowlania gify*)
```

```
frameRate = 50; (*klatek na sekundę*)  
slowMo = 0.25; (*Procent "rzeczywistej szybkości"*)  
czasGifa = 10; (*s*)  
czasSym = slowMo * czasGifa; (*Czas symulacji*)  
dt = slowMo * (1 / frameRate); (*Czas trwania jednej klatki*)
```

```
tabGif = plotAnim[#] & /@ Range[t0 + 0.001, t0 + czasSym - 0.001, dt];
```

```
file := "5_00Hz_" <> ToString[h] <> "px_" <> ToString[frameRate] <>  
  "fps_" <> ToString[slowMo] <> "slowMo_";
```

```
format := (*.EPS*) ".avi";
```

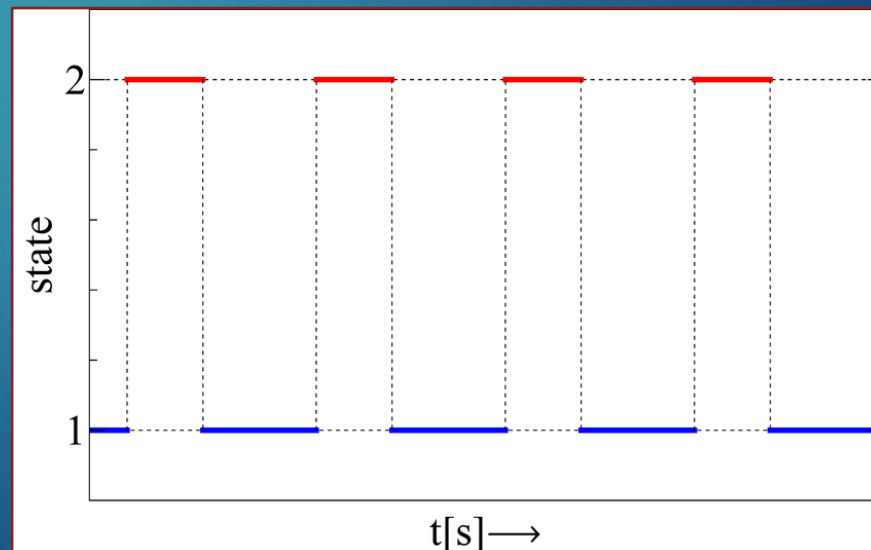
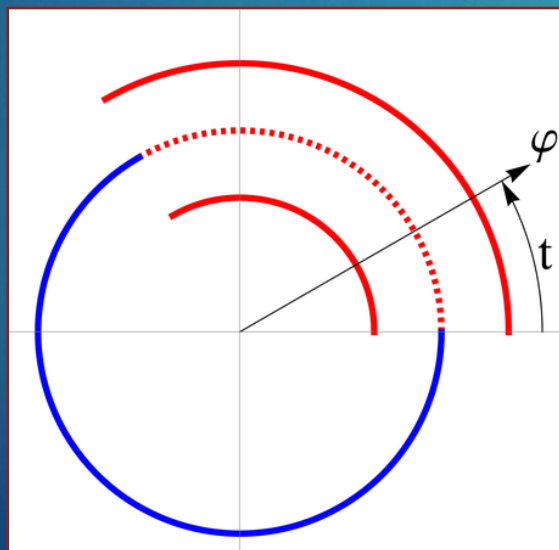
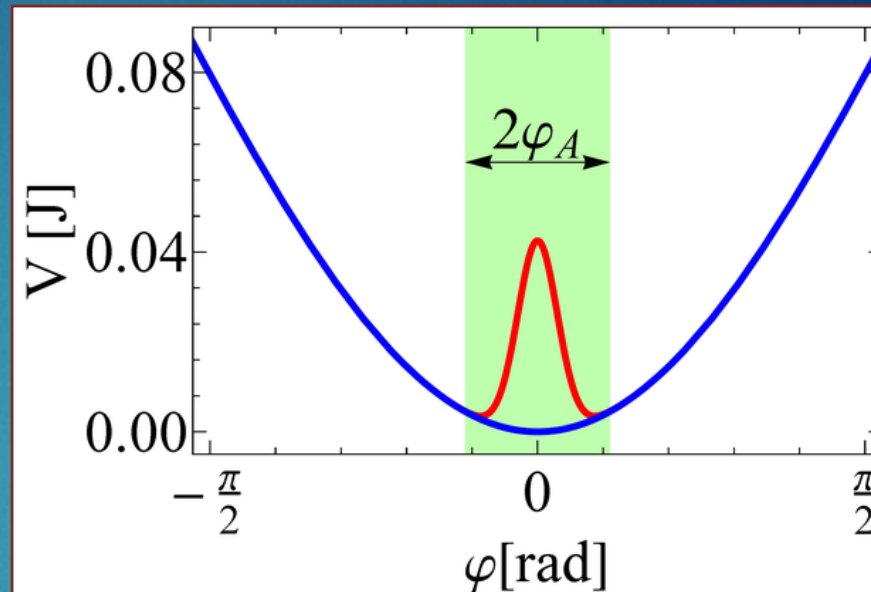
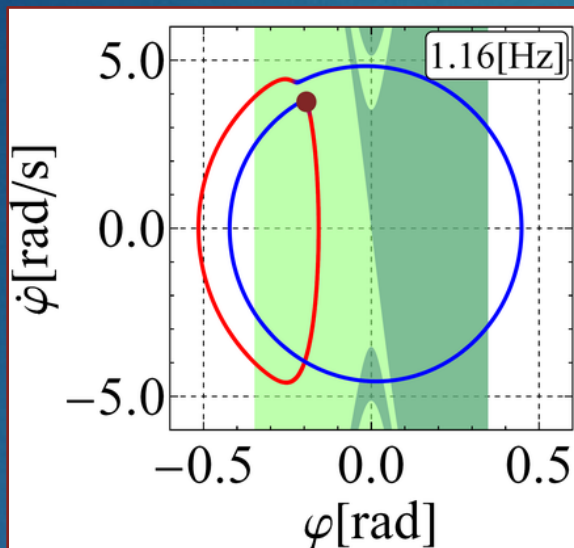
```
fileName := StringJoin[{file, format}]
```

```
Export[NotebookDirectory[] <> "\\Animacje\\" <> fileName, tabGif,  
  "FrameRate" -> frameRate
```

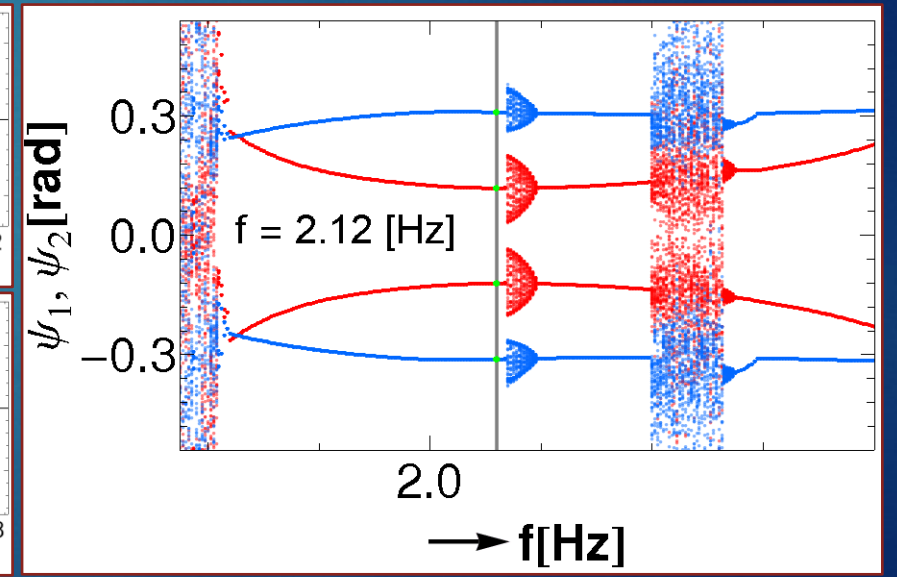
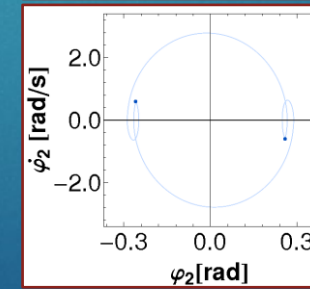
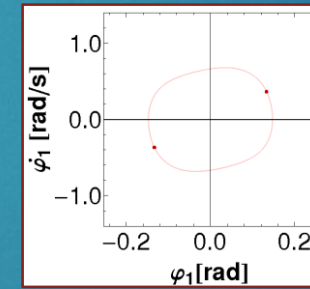
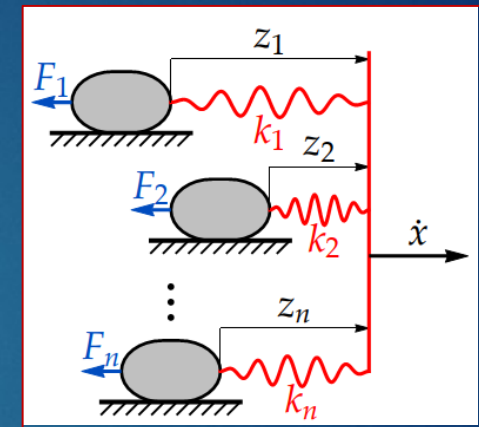
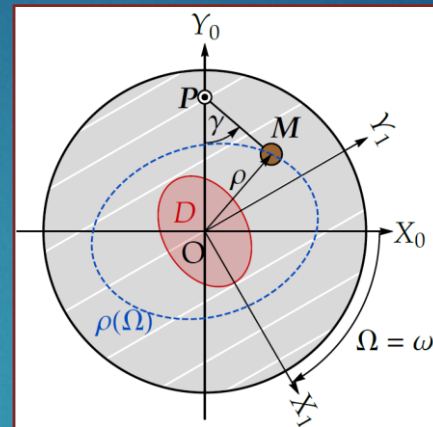
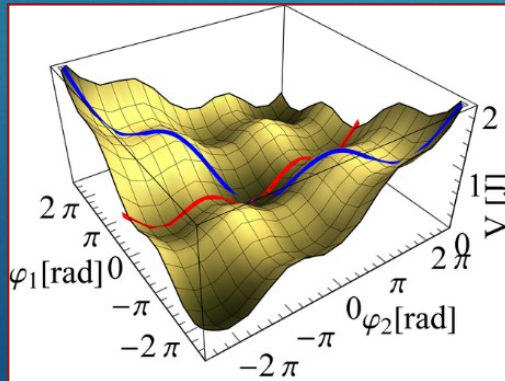
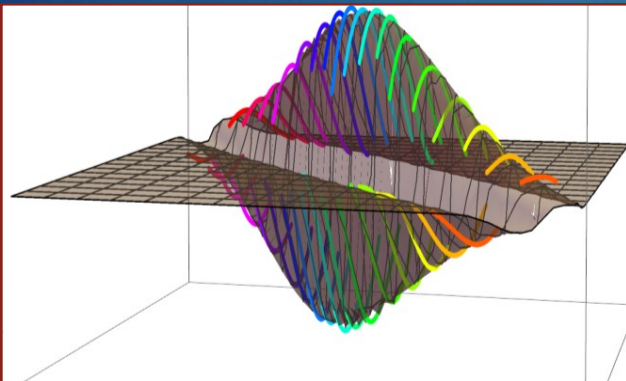
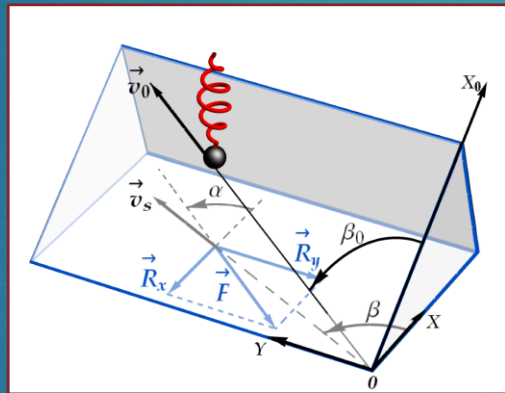
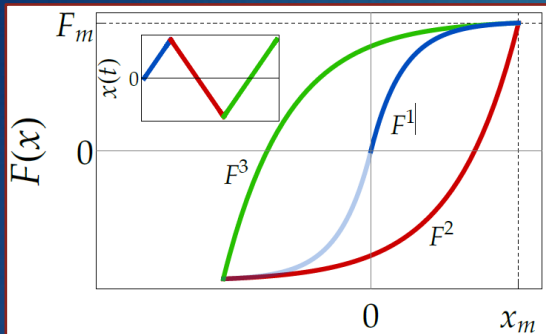
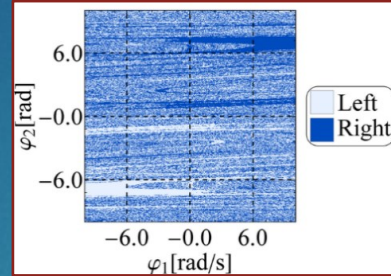
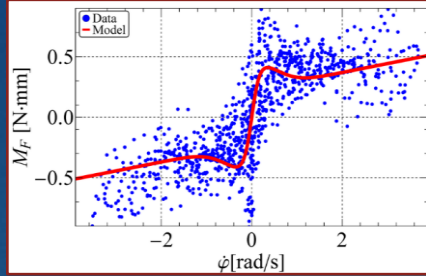
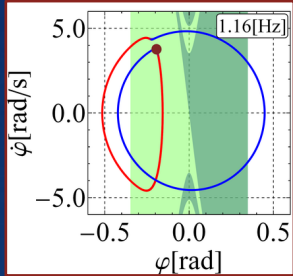
```
]
```



Prezentacja wyników – kod kolorów



Podsumowanie



Dziękuję za uwagę!